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NE2109-Reactor Theory and Analysis

Software Engineering (Dhaka University)

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Question Bank

Sunday, October 1, 2023 10:30 PM

2016

1-2016

a. Define laboratory system and CM system with respect to a collision between two particles. What is logarithmic energy decrement per collision? State the significance of this term. [5]

b. Obtain the expression for 'ξ' and show that a good approximate for 'ξ' is: [6]

$$\xi = \frac{2}{A + \frac{2}{3}}$$

c. If the average fractional energy loss per collision in Be nuclei is 0.209, what is the energy of 2 MeV neutrons after 50 collisions? [3]

2-2016

a. The symbol 'σ ' stands for the cross section per nucleus. Obtain the expression for σ to be $= \frac{r}{N_0 I V}$, the symbols carry the usual meanings. Demonstrate the fact that the expression has the dimension of an 'area'. [5]

b. Calculate and compare the collision and absorption mean free paths for neutrons in graphite. What is your comment? Given the values: [3]

$\sigma_s = 4.8$ barns, $\sigma_a = 3.2 \times 10^{-3}$ barns, $\rho = 2.25 gm/cm^3$, M for graphite is 12.

c. Distinguish four principal regions of neutron energies and discuss the energy dependence of neutron cross sections. [6]

3-2016

a. Derive the steady state diffusion equation for mono-energetic neutrons and explain the meaning of different terms in the equation. [6]

b. Discuss the boundary conditions while solving neutron diffusion equation. [2]

c. Solve the steady state neutron diffusion equation from a point source emitting Q thermal neutrons/sec, located in a homogeneous moderator of infinite extension. [6]

4-2016

a. Explain microscopic and macroscopic cross-sections. Obtain the relationship between macroscopic cross section and mean free path, λ. [4.5]

b. Define 'sdp' and 'mr'. Show that for a moderator of more than once element 'mr' is:

$mr = \frac{\xi_1 \Sigma_s^{(1)} + \xi_2 \Sigma_s^{(2)} + \cdots}{\Sigma_a^{(1)} + \Sigma_a^{(2)} + \cdots}$, the symbols have the usual meaning. [4.5]

c. Explain why energy is liberated during fission and fusion process. [3].

d. Explain why thermal neutrons can fission ^{235}U and only fast neutrons can fission ^{238}U. [2]

5-2016

a. Define excess reactivity, reactor period and e-folding period. [3]

b. Discuss the conditions which affect the reactivity. What is temperature coefficient, $\frac{d\rho}{dT}$? Express $\frac{d\rho}{dT}$ as a sum of partial contributions due to changes in the nuclear cross section σ, due to the change in the densities of the materials, d and due to change, in the B^2 (buckling) of the reactor. [5]

c. Elaborate the effect of temperature coefficient, $\frac{d\rho}{dT}$ on three temperature dependent quantities, k_∞, τ_0, and L^2. [6]

6-2016

a. What is the difference between radiative capture and fission? [3]

b. What is the fission product poisoning? Explain briefly Samarium poisoning with reactor operating time and different reactor power [6]

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c. Explain about the role of the moderators in nuclear reactors. [2]
d. How many fissions per second occur in a 3MW Uranium reactor? Assume 200MeV are released per fission $\left(1\text{eV} = 1.6 \times 10^{-19}\text{J}, 1\text{J} = 1.\text{Ws}\right)$. [3]

7-2016

a. Explain four factor formula for homogeneous reactor systems. What is the difference between four factor and six factor formulae? [4+2]
b. What is delayed neutrons? Why are delayed neutrons so important on reactor control? $[1 + 3]$
c. What are the differences between reactor material buckling and geometrical buckling? [4]

8-2016

a. Discuss the fuel temperature coefficient and why it is negative? [3+2]
b. Why water-moderated reactors are designed in an under moderated condition? [4]
c. Define shutdown margin. A reactor has a shutdown margin of $5.00\$$. What is k_{eff} ? $[2 + 3]$

2017

1-2017

a. Define thermal hardening. Why is the energy distribution of neutrons in a nuclear reactor not exactly Maxwellian? [3]
b. Explain why U-235 fissions with thermal neutrons and U-238 fission only with fast neutrons. [2]
c. Cite the difference between microscopic and macroscopic cross section. When does a neutron nucleus interaction show resonance peaks in the cross section versus energy curve? [3]
d. Draw the variations of the fission cross sections of U-235, U-238, and Pu-239 with neutron energy in the same graph and hence explain different regions. [4]
e. A reactor operating at a flux level of $3 \times 10^3\ neutrons/cm^2 sec$ contains 10^{20}atoms of U-235 per cm^3. The reaction rate is $1.29 \times\ 10^{12}\ fission/cm^3 sec$. Calculate Σ_f and σ_f. [2]

2-2017

a. Explain the effect that moderator temperature changes will have on the following factors: [3]
    i. Thermal utilization factor
    ii. Resonance escape probability
    iii. Fasti non-leakage probability
    iv. iv. Thermal non-leakage probability
b. What is the difference between k_{eff} and k_∞. Cite the condition for subcritical, critical and supercritical state of a nuclear reactor. [2]
c. Draw the variation of reproduction factor for U-233, U-235, and Pu-239 with neutron energy in the same graph. [2]
d. Draw the variations of the fission cross sections of U-235, U-238, and Pu-239 with neutron energy in the same graph and hence explain different regions. [2]
e. Calculate k_∞ for a homogeneous, natural uranium-graphite moderated assembly which contains 300 moles of graphite per mole of uranium. Assume natural uranium to contain one part of U^{235} to 139 parts of U^{238} and use these constant: [5]

Natural Uranium	Graphite
$\sigma_{\text{a(U)}} = 7.68\text{b}$ $\sigma_{\text{s(U)}} = 8.3\text{b}$	$\sigma_{\text{a(M)}} = 0.0032\text{b}$ $\sigma_{\text{s(M)}} = 4.8\text{b}$ $\xi = 0.158$

3-2017

a. Rank the following moderator materials [2]
    $\text{H}_2\text{O}, \text{D}, \text{O}, \text{Be}, \text{B}, \text{C}$
b. Define effective delayed neutron fraction. How it is related to the prompt criticality of the reactor.
c. Is it possible to make a reactor critical using only fertile material as fuel? If not, explain why.
d. Consider neutron slowing down in an infinite non absorbing medium. Show that the time

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required for the neutrons to slow down from an initial energy E_0 to final energy E_t is $T = \frac{(2m)^{1/2}}{\xi\Sigma_s}\left[\frac{1}{E_t^{1/2}} - \frac{1}{E_0^{1/2}}\right]$, where, m = mass of a neutrons, ξ = average logarithmic energy decrement per collision and Σ_s = macroscroscopic scattering cross section

e. Calculate the slowing down time in graphite for neutrons starting with an initial energy of $2\ MeV$ and terminating at thermal energies $(0.0253eV)$.

4-2017

a. Why is neutron moderation necessary although U^{235} undergo fission with fast neutrons?
b. Why is boron not used as a moderator in spite of its higher average logarithmic energy decrement per collision than graphite?
c. A neutron of initial kinetic energy of E_i scattered at an angle of θ with a final kinetic energy of E_f from a target nucleus of mass number A. Considering the collision as elastic show that the kinetic energy of the neutron after the collision is

$$E_f = \frac{E_i}{(A+1)^2}\left[\cos\theta + \sqrt{A^2 - \sin^2\theta}\right]^2$$

d. Using the above equation, show that the minimum final energy of the neutron is

$$E_f^{min} = \alpha E_i;\ \ \alpha = \left(\frac{A-1}{A+1}\right)^2$$

and hence show that maximum fractional energy loss possible is

$$\left(\frac{\Delta E}{E_i}\right)_{max} = (1-\alpha)$$

e. Calculate the average energy loss for neutrons that have made one collision with carbon nuclei, using $\xi = 0.158$ for carbon.

5-2017

a. What are the boundary conditions that are taken in the derivation of diffusion equation?
b. Derive the steady state diffusion equation.
c. Consider an infinite planner source emitting S neutrons per cm^2/sec in an infinite diffusing medium. The flux is defined as $\varphi = \varphi(x)$, where x is the distance from the plane. Show that the diffusion equation and the neutron flux are given by $\frac{d^2\varphi}{dx^2} - \frac{1}{L^2}\varphi \cdot 0$ and $\varphi(x) = \frac{SL}{2D}e^{-\frac{|x|}{b}}$ respectiveiy, where $L^2 = \frac{D}{\Sigma_a}$.

6-2017

a. Define the following terms:
   i. Burnable poison
   ii. Non-burnable poison
   iii. Chemical shim
b. State two reasons why fixed non-burnable neutron poisons are used in reactor cores with an example of a material used as a fixed non-burnable neutron poison.
c. Describe how samarium-149 is produced and removed from the reactor core during reactor operation.
d. Describe how samarium-149 concentration changes following a reactor shutdown from steady-state conditions.

7-2017

a. Show that, "the square of the diffusion length of a mono-energetic neutron is equal to one-sixth of the mean square distance from the source to the point at which the neutron is absorbed by a nucleus".
b. Derive the fundamental equations of reactor kinetics for the point reactor model.
   i. $\frac{dn}{dt} = k(\rho - \beta)\frac{n}{l} + \sum_{i=1}^{6}\lambda_i C_i$
   ii. $\frac{dC_i}{dt} = k\frac{\beta_1 n}{l} - \lambda_i C_i$
c. What is prompt critical? Find the condition for prompt criticality from the question (b).

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8-2017

a. Explain the mechanism for production of delayed neutrons.
b. List three desirable characteristics of a moderator. Define moderating ratio.
c. What do you mean by In-hour equation? Write down the In-hour equation and explain each terms.
d. Define doubling time in reactor kinetics.

2018

1-2018
2-2018
3-2018
4-2018
5-2018
6-2018
7-2018
8-2018

2019

1-2019

a. State and explain Fick's law for neutron diffusion. What are the underlying assumptions?
b. What are the boundary conditions that are taken in the derivation of diffusion equation? Consider an infinite planner source emitting S neutrons per $\mathrm{cm}^2/\mathrm{sec}$ in an infinite diffusing medium. The fux is defined as $\varphi = \varphi(x)$, where x is the distance from the plane. Show that the diffusion equation and the neutron flux are given by $\frac{d^2\phi}{dx^2} - \frac{1}{L^2}\varphi = 0$ and $\varphi(x) = \frac{SL}{2D}e^{-\frac{|x|}{L}}$ respectively, where $L^2 = \frac{D}{\Sigma_a}$.
c. The neutron flux in a bare spherical reactor of radius $50\ \mathrm{cm}$ is given by

$$\Phi = 5 \times 10^{13}\frac{\sin 0.0628\, r}{r}\ \mathrm{neutrons/cm^2 sec}$$

where r is measured from the center of the reactor. The diffusion coefficient for the system is $0.80\ \mathrm{cm}$.
   i. What is the maximum value of the flux in the reactor?
   ii. Calculate the neutron current density as a function of position in the reactor.
   iii. How many neutrons escape from the reactor per second?

2-2019

a. What is group diffusion method? Why is the one speed diffusion equation invalid for thermal nuclear reactor?
b. Describe multiplying medium and non-multiplying medium with examples. Draw a typical spatial neutron flux distribution for both the media.
c. The neutron flux for a critical spherical reactor of radius R is,

$$\Phi = \frac{A\sin\left(\frac{\pi r}{R}\right)}{r}$$

where "A" is a constant that determines the magnitude of the flux, Given the recoverable energy is E_M, joules per fission. Derive an expression for "A" in terms of operating power "P" at the reactor.
d. Calculate k_∞ for a homogeneous, natural uranium-graphite moderated assembly which contains 300 moles of graphite per mole of uranium. Assume natural uranium to contain one part of U^{235} to 139 parts of U^{238} and use these constant:

Natural Uranium	Graphite
$\sigma_{a(U)}$ = 7.68b $\sigma_{s(U)}$ = 8.3b	$\sigma_{a(M)}$ = 0.0032b $\sigma_{s(M)}$ = 4.8b ξ = 0.158

3-2019

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a. Explain the effects of heterogeneous arrangement of fuel and moderator on the following factors:

Fast Fission Factor	Resonance Escape Probability
Thermal Utilization factor	Reproduction Factor

b. Explain why is the diffusion equation invalid in the following system:
Absorber Surfaces, Core Moderator Surfaces

c. Consider neutron slowing down in an infinite non absorbing medium. Show that the time required for the neutrons to slow down from an initial energy E_0 to final energy E_t is

$$T = \frac{(2m)^{1/2}}{\sigma\Sigma_c}\left[\frac{1}{\varepsilon_t^{1/2}} - \frac{1}{\varepsilon_v^{1/2}}\right]$$

where, m = mass of a neutron, ξ = average logarithmic energy decrement per collision and Σ_s = macroscroscopic scattering cross section

d. Calculate the critical size and critical mass for a thermal reactor employing ^{235}U and graphite in an atomic ratio of $1:10^5$. Assume cubical shape with geometrical buckling of 0.018 cm. The required data are given below:

$$\sigma_{ag} = 0.003b; \lambda_{sg} = 2.60cm;\ \rho_g = 1.62\frac{g}{cm^3};\ \rho_u = 18.7\frac{g}{cm^3}$$

4-2019

a. Why a startup neutron source may be required for a reactor?
b. Describe how xenon- 135 is produced and removed from the reactor core during reactor operation.
c. How xenon- 135 concentration changes following a reactor shutdown from steady state conditions? Explain.
d. Explain what is meant by the following terms:
   i. Xenon precluded startup
   ii. Xenon dead time
e. What is power tilt?

5-2019

a. Derive the following In-hour equation:

$$\rho = \frac{l_p\omega}{1+l_p\omega} + \frac{\beta}{\omega+\lambda}\frac{\omega}{1+l_p\omega}$$

Hence find the analytical solution for, $l_p\omega \ll 1$.

b. Starting from the fundamental equations of reactor kinetics for the point reactor model.

   i. $l_p\frac{d\emptyset}{dt} = k_\infty\emptyset(1-\beta) + \frac{\lambda C}{\Sigma_4} - \emptyset$

   ii. $\frac{dC}{dt} = \beta k_\infty\Sigma_a\emptyset - \lambda C$

And the solutions found from the section (a), show that the full approximate solution for the flux can be written as:

$$\frac{\emptyset}{\emptyset_0} = \left(\frac{\beta(1-\rho)}{(\beta-\rho)}\right)e^{\left(\frac{\lambda p}{\beta-p}\right)t} - \frac{\rho(1-\beta)}{(\beta-\rho)}e^{-\left(\frac{\beta-\rho}{l_p}\right)}$$

6-2019

a. Explain the difference between a grey neutron absorbing material and a black neutron absorbing material.
b. Classify control rods based on the purpose and briefly explain them.
c. Describe the shape of a typical differential and integral control rod worth curve and explain the reason for the shape.
d. For the differential rod worth data given below, construct differential and integral rod worth curves.

Interval (inches)	Reactivity inserted (pcm)
0 to 2	10
2 to 4	20

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4 to 6	40
6 to 8	60
8 to 10	60
10 to 12	40
12 to 14	20
14 to 16	10

7-2019

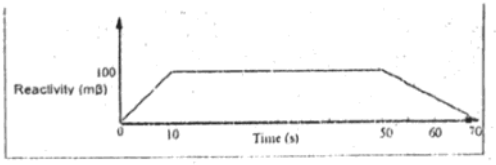
a. What do you understand by total control rod worth, SDM and core excess reactivity? Establish a relationship among them.
b. Classify control rods based on the purpose and briefly explain them. What does the SDM express in both $ and p.
c. How the worth of a control rod is measured by positive period method? Explain the steps.
d. Define the term reactivity.

8-2019

a. Why is the void coefficient of reactivity for CANDU positive?
b. Describe the reactivity feedback (i.e. positive or negative) of the following effects:

Fuel Breeding	Burnable Poison
Thermal Hardening	Fuel Burnup

c. What are the advantages and disadvantages of chemical shim over fixed burnable poisons?
d. Show graphically how the e-folding time and power of the nuclear reactor change in which the reactivity insertion has the following shape



2020

1-2020

a. State Fick's law for neutron diffusion and write down its application in reactor case.
b. Consider a point source emitting S neutrons/sec isotropically in an infinite medium. If the source point is taken to be at the center of a spherical coordinate system, the flux obviously only depends on r. Show that the neutron flux at a distance r from the source is

$$\varphi = \frac{Se^{\frac{r}{L}}}{4\pi Dr}$$

Where, L = Diffusion Length and D = Diffusion Coefficient

2-2020

a. What's the importance of delayed neutrons in reactor operation.
b. Explain the advantages and disadvantages of chemical shim over fixed burnable poisons.

3-2020

a. Define the temperature coefficient of reactivity. Explain why a negative temperature coefficient of reactivity is desirable?
b. Show that, "the square of the diffusion length of a mono-energetic neutron is equal to one-sixth of the mean square distance from the source to the point at which the neutron is absorbed by a nucleus".

4-2020

a. What do you understand by total control rod worth, SDM and Core excess reactivity? Establish a relationship among them.
b. A reactor has a total worth of *$12.50* and measured core excess reactivity of *$6.80*. What is the SDM expresses in both *$* and ρ.

5-2020

a. classify control rods based on the purpose and briefly explain them.
b. Describe the shape of a typical differential and integral control rod worth curve and explain the reason for the shape.

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6-2020

a. Explain neutron shadowing and disadvantage factor.
b. A monoenergetic neutron beam of intensity I is incident on a sheet of U-235 of area A and thickness t. Prove that the macroscopic fission cross section Σ_f can be written as

$$\Sigma_f = \frac{\dot{R_v}}{I}$$

where R_v is fission rate per unit volume.

7-2020

a. Cite the neutron transport equation and explain different terms.
b. A reactor core contains enriched uranium as fuel and beryllium oxide as moderator ($\xi = 0.17$ and $\Sigma_s = 64\ m^{-1}$ at a neutron energy of $1\ eV$). The thermal neutron flux is $2 \times 10^{16}\ neutrons/m^2 sec$ and Σ_a for these neutrons in the fuel, is $0.5\ m^{-1}$; for each thermal neutron absorbed, 1.7 fission neutrons are produced. Neglecting all losses during slowing down, estimate the neutron flux at $1\ eV$ per unit lethargy interval.

8-2020

a. Explain prompt drop and prompt jump schematically.
b. Consider a light water-cooled graphite-moderated thermal reactor fueled with low-enriched uranium (LEU) which operates with burnup. Explain the causes of the results in the cases of
   i. a positive coolant void coefficient and
   ii. a positive moderator temperature coefficient

2021

1-2021

a. Define neutron cross section and briefly explain different region of a typical neutron cross section against neutron energy graph.
b. What is mean free path? Show that mean free path is inversely proportional to the macroscopic cross section.
c. Write down the energy dependence cross section for slow neutrons and hence neglecting the scattering cross section and using the values for the thermal neutron cross sections for ^{235}U, calculate $\frac{\Gamma_\gamma}{\Gamma}$ and $\frac{\Gamma_f}{\Gamma}$. Capture, fission and scattering cross section of ^{235}U for $0.0253\ eV$ neutrons are as follows: $101\ b$, $582\ b$ and $10.0\ b$ respectively.

2-2021

a. Why the average energy is no longer a good measure for the average neutron behavior? Establish an expression of logarithmic energy decrement per collision (ξ) in terms of atomic mass number (A).
b. Calculate the sdp and the mr for D_2O, using these numerical values:

$$\rho_{D_2O} = 1.10 \frac{g}{cm^3};\ \sigma_s = 10.6 \text{ barns},\ \sigma_a = 0.001 \text{ barn}$$

c. What is slowing down density? In the absence of neutron absorption, show that slowing-down neutron flux per unit energy is inversely proportional to the energy E.
d. Calculate the slowing-down time in graphite for neutrons starting with initial energy of $2\ MeV$ and terminating at thermal energies $(0.025\ eV)$. Given that, $\Sigma_{s(graphite)} = 0.385\ cm^{-1}$.

3-2021

a. What do you understand by the term "Thermal Utilization Factor"? Develop an equation for thermal utilization factor in terms of reaction rate and hence simplify it for homogenous nuclear reactor.
b. Calculate the reproduction factor for a reactor that uses 10% enriched uranium fuel. Necessary data for this calculation are provided below.

	U − 235	U − 238
σ_c(barns)	101	2.7
σ_f(barns)	582	0
σ_s(barns)	10	8.3
N $\left(\frac{\text{atoms}}{\text{cm}^3}\right)$	4.82×10^{22}	4.35×10^{22}

Number of neutrons released per fission is 2.42.

c. Why is heterogeneous nuclear reactor more preferable than homogeneous?
d. Briefly discuss the effect of heterogeneous arrangement of fuel and moderator on p, f, and ε.

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e. Calculate "f" for a heterogeneous lattice made up of cylindrical uranium metal rods of *3 cm* diameter which are spaced at regular intervals of *18 cm* in graphite. Assume a thermal disadvantage factor of *1.6* and these constants for the materials.

Uranium density, ρ_U = *18.7 gcc*	Graphite density, ρ_m = *1.61 gcc*
σ_{aU} = *7.68 $barns$*	σ_{am} = *0.0045 $barns$*

4-2021

Neutrons in a nuclear reactor actually have a distribution in energy; this distribution must be accounted for in the diffusion equation. Assume that entire range of neutron energy is divided into N energy intervals as indicated in figure. Now, consider the g^{th} group neutrons and answer the following questions. (Note: the most energetic group neutrons is denoted by g = 1; the least energetic group has g = N)

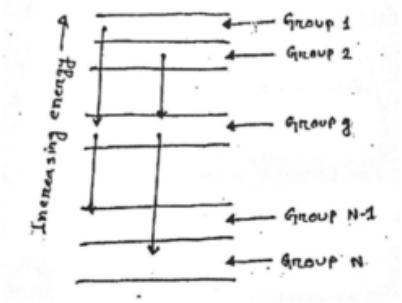

Figure: Energy groups for a group diffusion calculation

a. Derive the steady state diffusion equation for g^{th} group neutrons.
b. Why at least two groups of neutrons are important for reactor calculations?
c. Starting from (i) obtain the group diffusion equation for fast neutrons emitting from a point S in an infinite uniform moderator and solve it.
d. Microscopic three-group cross-sections (in barns) of sodium for calculations of sodium-cooled fast reactors are given in table. Suppose that at some point in such a reactor the three group fluxes are $\Phi_1 = 6 \times 10^{14}$, $\Phi_2 = 1 \times 10^{15}$, and $\Phi_3 = 3 \times 10^{15}$ Calculate for this point the number of neutrons (a) absorbed per cm^3/sec in sodium; (b) scattered per cm^3/sec from the first group to the second as the result of collisions with sodium. The sodium is at normal density with atom density $0.02541 \times 10^{24} \frac{atoms}{cm^3}$.

g	Energy range, MeV	σ_γ	σ_g	$\sigma_{g\to g+1}$	$\sigma_{g\to g+2}$
1	1.35 $\to \infty$	0.0005	2.0	0.24	0.06
2	0.4 – 1.35	0.001	3.2	0.18	
3	0 – 0.4	0.001	3.7		

5-2021

a. Define reflector savings. What effect does the reflector have on the power profile of the reactor?
b. Consider neutron slowing down in an infinite non absorbing medium. Show that the time required for the neutrons to slow down from an initial energy E_0 to final energy E_t is

$$T = \frac{(2m)^{1/2}}{\xi \Sigma_s}\left[\frac{1}{E_t^{1/2}} - \frac{1}{E_0^{1/2}}\right]$$

where, m = mass of a neutron, ξ = average logarithmic energy decrement per collision and Σ_s = macroscopic scattering cross section

c. Prove that the critical mass of an infinite homogenous reactor of volume *V* can be written as

$$m_F = Z\frac{\sigma_{aM}(E_0)M_F}{g_{aF}(T)\sigma_{aF}(E_0)M_M}M_M$$

Where,σ_{aM} = absorption cross section of moderator, σ_{aF} = absorption cross section of fuel, M_F = mass of fuel,M_M = mass of moderator,$g_{aF}(T)$ = non $\frac{1}{v}$ factor, $Z = \frac{\Sigma_{aF}}{\Sigma_{aM}}$.

d. A spherical tank with a radius of *40 cm* is filled with a homogeneous mixture of $U-235$ and light

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water. The mixture has a moderator-to-fuel ratio $N_{H_2O}/N_{U235}$of *800*.

i. What is $k_\infty$of the mixture?

ii. What is k_{eff} for this bare core?

&σ_f^{235} = 505.9b, σ_a^{235} = 592.6b, ϑ = 2.43, σ_a^{Water} = 0.6644, L^2_{water} = 8.12cm^2, τ = 27cm^2

6-2021

a. Explain transport mean free path *(λ_{tr})*. The steady state neutron transport equation can be written as

$$\hat{\Omega}.\bar{\nabla}\Psi(\vec{r}, E, \hat{\Omega})\& + \Sigma_t(\vec{r}, E)\Psi(\vec{r}, E, \hat{\Omega})$$
$$= \int_0^\infty dE' \int_{4\pi} \Sigma_s\left(\vec{r}, E' \to E, \hat{\Omega}' \to \hat{\Omega}\right)\Psi\left(\vec{r}, E', \hat{\Omega}'\right) d\Omega$$
$$+ \frac{\chi(\vec{r}, E)}{4\pi}\int_0^\infty dE' \int_{4\pi} \vartheta\Sigma_f(\vec{r}, E')\Psi\left(\vec{r}, E', \hat{\Omega}'\right) d\hat{\Omega} + \frac{Q(\vec{r}, E)}{4\pi}$$

where, $\vec{r}$, E, $\hat{\Omega}$, Ψ, Σ denote the position, energy, direction of flight, angular flux and macroscopic cross section of neutrons respectively. The average number of neutrons per fission is ϑ and $\chi(\vec{r}, E)$ is the probability that a fission neutron is emitted at $\vec{r}$ with energy E. The system also contains an independent neutron source with source strength Q.

i. Explain each of the terms of this equation.

ii. Write multigroup transport equation

b. Define Fermi age *(τ)*. Prove that τ is equal to one-sixth of the square of the average distance (in all dimension) between the neutron's birth point (as a fast neutron) and the point, where it has become thermalized.

c. Calculate k_∞ for a homogeneous, natural uranium-graphite moderated assembly which contains *300* moles of graphite per mole of uranium. Assume natural uranium to contain one part of U^{235} to 139 parts of U^{238} and use these constants:

Natural Uranium	Graphite
σ_{aU} = 7.68 *b* σ_{sU} = 8.3 *b*	σ_{aM} = 0.0032 *b* σ_{sM} = 4.8 *b* ξ = 0.158

7-2021

a. What do you understand by the term bare reactor?

b. Starting from one group time dependent diffusion equation derive the one group reactor equation.

c. Solve the reactor equation for the infinite bare slab reactor of thickness *"a" ($a \gg d$; d* is extrapolated distance from the slab face, where flux vanishes completely) as shown in figure. Here,

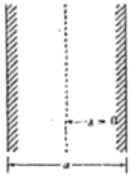

Figure: Infinite bare slab reactor

d. A fast reactor assembly consisting of a homogeneous mixture of ^{239}Pu and sodium is to be made in the form of a bare sphere with geometrical buckling $B_1 = \left(\frac{\pi}{R}\right)^2$. The atom densities of these constituents are N_F = *0.00395 × 10^{24}* for the ^{239}Pu and N_s = *0.0234 × 10^{24}* for the sodium. Estimate the critical radius R_c of the assembly. Necessary data are provided below:

$\sigma_{a(Pu)}$ = 2.11 barns, $\sigma_{tr(Pu)}$ = 6.8 barns, $\sigma_{a(\text{Na})}$ = 0.0008 barn, $\sigma_{\text{tr(Na)}}$ = 3.3 barns

8-2021

a. What are the assumptions of point reactor kinetics? Explain prompt drop and prompt jump schematically.

b. Why is Xe-135 and Sm-149 important in reactor analysis? Prove that the equilibrium concentrations of Pm-149 and Sm-149 are given by

$P_{eq} = \frac{\gamma_{Nd}\bar{\Sigma}_f\varphi}{\lambda_{Pm}}$	$S_{eq} = \frac{\gamma_{Nd}\bar{\Sigma}_f}{\bar{\sigma}_a^{Sm}}$

where $\gamma, \lambda, \varphi, and\ \sigma$ denote fission yield, decay constant, flux and cross section respectively and $\bar{\Sigma_f}$ is the average macroscopic fission cross section of fuel.

c. Many phenomena produce reactivity feedback. Within orders of magnitude, over what time interval would you expect the following reactivity feedback causes to take effect?

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i. Temperature increase in the fuel
ii. Temperature increase in the moderator/coolant,
iii. Xe-135 increase,
iv. Fuel burnup,
v. Boiling in the core
vi. increased coolant flow through the core

d. Following a reactor scram, in which all the control rods are inserted into a power reactor, how long is it before the reactor power decreases to *0.0001* of the steady-state power prior to shut down?

2022

1-2022

a. Consider the elastic collision of neutrons with nucleus of mass number A. A neutron of initial kinetic energy of E_i is scattered at an angle of θ_C (measured in CM frame) with a final kinetic energy of E_f. Prove that
   i. If both E_i and E_f are measured in the Lab frame, the kinetic energy after collision is given by [5]

$$E_f = E_i \frac{A^2 + 2A\cos\theta_C + 1}{(A+1)^2}$$

   For isotropic scattering, the energy transfer probability can be written as

$$P(E_i \to E_f) = \frac{1}{(1-\alpha)E_i}$$

   Here, $\alpha = (A-1)^2/(A+1)^2$

b. Why is lethargy more convenient for neutron slowing down calculation? Prove that the slowing down density in an infinite hydrogen moderator without absorption is given by [4]

$$q(u) \approx \Sigma_s(u)\emptyset(u)$$

   where, $\Sigma_s(u)$ and $\emptyset(u)$ are the scattering cross section and neutron flux respectively in terms of lethargy variable.

c. Why is fast neutron spectrum desirable in the design of breeder reactor? Consider the scattering of a neutron from both the bound and free atom of mass number A. Show that the scattering cross sections in case of bound atom is given by [3]

$$\sigma_S^{Bound} = \sigma_S^{Free} \frac{(1+1/A)^2}{(1+1/A_{eff})^2}$$

d. A soluble salt of fully enriched uranium is dissolved in water to make a solution containing 1.5×10^3 atoms of $U-235$ per molecule of water. [2]
   i. Explain why $\varepsilon p \approx 1$ for this solution.
   ii. What is k_∞ for this solution? Neglect any neutron absorption by other elements in the uranium salt. $\sigma_f^{235} = 505.9b, \sigma_a^{235} = 592.6b, \nu = 2.43, \sigma_a^{Water} = 0.6644$

2-2022

a. What are the assumptions in neutron transport theory. The time dependent neutron transport equation can be written as [4]

$$\frac{1}{v(E)}\frac{\partial\Psi(\vec{r},E,\hat{\Omega})}{\partial t} + \hat{\Omega}.\bar{\nabla}\Psi(\vec{r},E,\hat{\Omega}) + \Sigma_t(\vec{r},E)\Psi(\vec{r},E,\hat{\Omega})$$
$$= \int_0^\infty dE' \int_{4\pi} \Sigma_n(\vec{r},E' \to E, \hat{\Omega}' \to \hat{\Omega})\Psi(\vec{r},E',\hat{\Omega}')d\Omega'$$
$$+ \frac{\chi(E)}{4\pi}\int_0^\infty dE' \int_{4\pi} \theta E_f(\vec{r},E')\Psi(\vec{r},E',\hat{\Omega}')d\hat{\Omega}' + \frac{Q(\bar{r},E)}{4\pi}$$

   where, $\vec{r}, E, \hat{\Omega}, \Psi, \Sigma$ denote the position, energy, direction of flight, angular flux and macroscopic cross section of neutrons respectively. The average number of noutrons per fission is θ and $\chi(E)$ is the probability that a fission neutron is emitted with energy E. The system also contains an independent neutron source with source strength Q.
   i. Simplify this equation for one speed approximation and isotropic scattering.
   ii. To numerically solve the neutron transport problem using algebraic equations,

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the spatial, angular, energy and time variables must be discretized. Explain Discrete Ordinate Method (S_N) and Spherical Harmonics Method (P_N) for the discretization of anguiar variables.

b. Cite boundary conditions for the steady-state neutron diffusion equation. Consider an infinite slab of thickness $2a$ that has an infinite planar source at its center emitting S neutrons per cm^2/sec. Prove that the flux at a given point in the slab is given by [4]

$$\varphi = \frac{SL}{2D}\frac{\sinh[(a+d-|x|)/d]}{\cosh(a+d)/L}$$

where, $D =$ diffusion coefficient, $L =$ diffusion Length, $d =$ extrapolated distance

c. The thermal flux in a bare cubical reactor is given approximately by the function: [6]

$$\emptyset_T(x,y,z) = A\cos\left(\frac{\pi x}{\tilde{a}}\right)\cos\left(\frac{\pi y}{\tilde{a}}\right)\cos\left(\frac{\pi z}{\tilde{a}}\right)$$

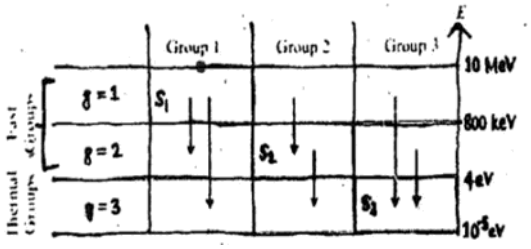
where A is a constant, a is the length of a side of the cube, $\tilde{a}$ is a plus $2d$, d is the extrapolation distance, and x, y, and z are measured from the center of the reactor. Derive expressions for the

i. thermal neutron current as a function of position in the reactor;
ii. number of thermal neutrons leaking per second from each side of the reactor;
iii. total number of thermal neutrons leaking per second from the reactor.

3-2022

a. What is group diffusion method? Why is the one speed diffusion equation invalid for thermal nuclear reactor?

b. Explain group collapsing of cross section. Using the following slowing-down neutron balance in three-group problem, write the neutron balance equation of each group.



c. Cite different methods for the numerical solution of diffusion equation. Consider the following multigroup diffusion equation

$$-\bar{\nabla}\cdot D_g\bar{\nabla}\varphi_g + \Sigma_{Rg}\varphi_g = \sum_{g=1}^{g-1}\Sigma_{sg\to g}\varphi_{\dot{g}} + \frac{1}{k}\chi_g\sum_{g=1}^{G}\vartheta_{\dot{g}}\Sigma_{fg}\varphi_g$$

where symbols have their usual meaning.

Write this equation in the following matrix form.

$$M\varphi = \frac{1}{k}F\varphi$$

Hence describe the strategy for reactor criticality calculation using source iteration method in a flowchart.

d. The three-group fluxes for a base spherical fast reactor of radius $R = 50$ cm are given by the following expressions:

$$\emptyset_1(r) = \frac{3\times10^{15}}{r}\sin\left(\frac{\pi r}{R}\right)$$
$$\emptyset_2(r) = \frac{2\times10^{16}}{r}\sin\left(\frac{\pi r}{R}\right)$$
$$\emptyset_3(r) = \frac{1\times10^{16}}{r}\sin\left(\frac{\pi r}{R}\right)$$

The group-diffusion coefficients are $D_1 = 2.2cm, D_2 = 1.7cm$, and $D_3 = 1.05cm$. Calculate the total leakage of neutrons $\left(\sum_{g=1}^{3} - D_g\nabla^2\emptyset_g\right)$ from the reactor in all three groups. [Ignore the extrapoiation distance.]

4-2022

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a. Define geometric and material buckling. Explain reactor criticality based on the concept of buckling. [2]
b. Show that, "the square of the diffusion length of a mono-energetic neutron is equal to one-sixth of the mean square distance from the source to the point at which the neutron is absorbed by a nucleus". [4]
c. Derive the flux distribution for a spherical reactor with radius R operating at power P by solving the one-group reactor equation. [5]
d. Determine the height to diameter ratio of a bare cylindrical reactor that will lead to the smallest critical mass. [3]

5-2022

a. What is thermal disadvantage factor? Prove that $\left(\mathrm{k}_\infty\right)_{\mathrm{hetero}} > \left(\mathrm{k}_\infty\right)_{\mathrm{homo}}$ for a system with same composition. [2]
b. Consider a non-critical system consisting of an infinite bare slab of thickness a. Using one group reactor model show that the transient flux of the system can be written as [5]

$$\varphi(\mathrm{x},\mathrm{t}) = \mathrm{A}_1 \exp\left(-\lambda_1 \mathrm{t}\right)\cos\left(\mathrm{B}_1 \mathrm{x}\right)$$

where, A_1 and λ_1 are constants, $\mathrm{B}_1 =$ buckling. Hence verify the following conditions.

$$\mathrm{B}_\mathrm{m}^2 > \mathrm{B}_\mathrm{g}^2 \rightarrow \lambda_1 < 0;$$
$$\mathrm{B}_\mathrm{m}^2 = \mathrm{B}_\mathrm{g}^2 \rightarrow \lambda_1 = 0;$$
$$\mathrm{B}_\mathrm{m}^2 < \mathrm{B}_\mathrm{g}^2 \rightarrow \lambda_1 > 0;$$

c. Using one-group theory, show that expressions for the flux and the condition for criticality for a spherical reactor of radius R with a reflector of thickness b is given by [5]

$$\emptyset_\mathrm{C}(\mathrm{r}) = \mathrm{C}\frac{\sin\left(\mathrm{B}_\mathrm{C}\mathrm{r}\right)}{\mathrm{r}}$$

$$\emptyset_\mathrm{R}(\mathrm{r}) = \mathrm{C}\sin\left(\mathrm{B}_\mathrm{C}\mathrm{r}\right)\frac{\sinh\left(\frac{\mathrm{R}+\mathrm{b}-\mathrm{r}}{\mathrm{L}}\right)}{\mathrm{r}\sinh\left(\frac{\mathrm{b}}{\mathrm{L}}\right)}$$

$$\mathrm{D}_\mathrm{C}\left[-\mathrm{B}_\mathrm{C}\mathrm{R}\cot\left(\mathrm{B}_\mathrm{C}\mathrm{R}\right)+1\right] = \mathrm{D}_\mathrm{R}\left[\frac{\mathrm{R}}{\mathrm{L}}\coth\left(\frac{\mathrm{b}}{\mathrm{L}}\right)+1\right]$$

Here, subscript C and R denote the core and reflector respectively, $\mathrm{B} =$ buckling and $\mathrm{L} =$ diffusion length.

d. What is reflector saving? Estimate the critical radius of a hypothetical bare spherical reactor with $\mathrm{L}^2 = 22072\ \mathrm{cm}$ and $\mathrm{k}_\infty = 2.315$. [2]

6-2022

a. Why is negative temperature coefficient of reactivity desirable in the design of nuclear reactor? Consider a nuclear reactor which is initially critical. A control rod is inserted suddenly into the core and the reactor falls subcritical. Using modified one group theory, prove that worth of the control rod is given by

$$\rho_\mathrm{w} = \frac{2\mathrm{M}^2\mathrm{B}_0\Delta\mathrm{B}}{1+\mathrm{B}_0^2\mathrm{M}^2}$$

Here, $\mathrm{M} =$ neutron migration area, $\mathrm{B}_0 =$ initial buckling, $\Delta\mathrm{B} = \mathrm{B}-\mathrm{B}_0$, $\mathrm{B} =$ buckling after rod insertion.

b. Why is the temperature coefficient $\left[\alpha_\mathrm{T}\left(\mathrm{P}_\mathrm{T}\right)\right]$ of thermal nonleakage probability $\left(\mathrm{P}_\mathrm{T}\right)$ negative? Prove that $\alpha_\mathrm{T}\left(\mathrm{P}_\mathrm{T}\right)$ can be written as

$$\alpha_\mathrm{T}\left(\mathrm{P}_\mathrm{T}\right) = -\frac{\mathrm{B}^2\mathrm{L}_\mathrm{T}^2}{1+\mathrm{B}^2\mathrm{L}_\mathrm{T}^2}\left[\alpha_\mathrm{T}\left(\mathrm{L}_\mathrm{T}^2\right)+\alpha_\mathrm{T}\left(\mathrm{B}^2\right)\right]$$

Here, $\mathrm{B}^2 =$ geometrical buckling and $\mathrm{L}_\mathrm{T}^2 =$ thermal diffusion area

c. Cite advantages of chemical shim. The thermal utilization of a certain rod-and-shim controlled PWR is 0.930 at startup (clean and cold). At this time, the total excess reactivity in the reactor is 20.5%. If the control rods are worth 8.5%, what is the minimum $\mathrm{H_3BO_3}$ concentration in ppm and g/ liter of unit-density water that is necessary to hold down the

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reactor-that is, to keep it from becoming critical?

d. Explain shutdown margin. A control rod is dropped into a critical, source-free, uranium fueled reactor and the asymptotic period of the resulting exponentially decreasing power is observed to be -200 s.
   i. What is the value of k of the reactor after the control rod drop?
   ii. What was the reactivity insertion in dollars?

   Delayed neutron faction $= 0.0065$, Avarage lifetime of delayed neutron $= 12.8$ sec

7-2022

a. Explain the importance of delayed neutrons in reactor operation.
b. Derive the fundamental equations of reactor kinetics for the point reactor model.

$$\frac{\mathrm{dn}}{\mathrm{dt}} = \mathrm{k}(\rho - \beta)\frac{\mathrm{n}}{\mathrm{l}} + \sum_{\mathrm{i}=1}^{6} \lambda_{\mathrm{i}} \mathrm{C}_{\mathrm{i}}$$

$$\frac{\mathrm{dC_i}}{\mathrm{dt}} = \mathrm{k}\frac{\beta_{\mathrm{i}} \mathrm{n}}{\mathrm{l}} - \lambda_{\mathrm{i}} \mathrm{C}_{\mathrm{i}}.$$

c. Define reactivity and reactor period, and explain how they are related using the in-hour equation.
d. Calculate the reactivity of a graphite-uranium reactor when its steady-state power operation is abruptly changed by adjusting the position of a control rod, resulting in a stable reactor period of 10.5 minutes. Given that, average mean life of delayed neutron groups, $\bar{\tau} = 13\mathrm{s}$.

8-2022

a. Explain the mechanism of xenon oscillation in a nuclear reactor, including the physical processes and factors that contribute to its onset.
b. Describe the production and removal mechanisms of Xenon-135 during the operation of a nuclear reactor and derive an expression for the equilibrium concentration of xenon.
c. Describe Samarium poisoning in the reactor and derive the equation for the equilibrium concentration of samarium.
d. Calculate the ratio between the equilibrium concentrations of xenon in a thermal neutron nuclear reactor and an epithermal neutron nuclear reactor with equivalent power, given the neutron flux and xenon absorption cross section values: $\Phi_T = 5 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^2 s}$, $\sigma_{xe}^{T} = 3 \times 10^{-18}$ cm^{-2} for the thermal reactor, and $\phi_{ET} = 10^{15} \frac{\text{neutrons}}{\text{cm}^2\ \text{s}}$, $\sigma_{Xe}^{ET} = 10^{-23}$ cm^{-2} for the epithermal reactor. Half life of $Xe-135$, $t_{1/2}^{Xe} = 9.14$ hours.

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1. Nuclear Reaction

Wednesday, September 13, 2023 11:30 PM

1.1 Introduction - why nuclear

- Nuclear power reactors, based on nuclear fission reactions, convert mass into energy following the famous $E = mc^2$ formula of Albert Einstein.
- Contrasting nuclear power with fossil fuels highlights significant differences in energy production, fuel consumption, and by-products. [exercise energy density calculation]

Aspect	Coal	Nuclear
Energy Production	Chemical reaction: $C + O_2 \rightarrow CO2$	Nuclear reaction: $n + {}^{235}U \rightarrow$ Fission
Energy Release	$\sim 4.0\ eV$ per carbon atom [$394\ kJ/mol$]	$\sim 200\ MeV$ per uranium atom
Energy Density [Specific energy]	~ 30 MJ/kg	$\sim 80 \times 10^6 MJ/kg$

  - $1000\ MWe$ power plant - Coal vs Uranium

Fuel Consumption	$\sim 10{,}000\ tons/day$ of coal	$\sim 20\ tons/yr$ of Uranium [1 ton U-235!]
Supply Patterns	Continuous supply of coal [~100 coal cars/day]	Periodic refueling (12-24 months)
Waste Products	Large quantities of ash [$\sim 500\ tons/day$]	Smaller quantities of highly radioactive waste [$10\ tons/yr$ with reprocessing]
GHG emission	Thousands of tons of CO_2 emissions.	No GHG emission

  - A typical pellet of Uranium fuel [$m \sim 7\ gm$] generates as much energy as
    - ~560 Liters of oil
    - ~480 cubic meters of natural gas
    - ~800 kg of coal

1.2 Nuclear Reaction Fundamentals

1.2.1. Notation

- Nuclide denoted as ${}^{A}_{Z}X$ where $A = Z + N$
  - e.g. ${}^{12}_{6}C$ or ${}^{235}_{92}U$
  - Isotopes, Isobars, Isotones?
- Shorthand for some common nuclides
  - ${}^{4}_{2}\mathrm{He} \Rightarrow \alpha$, ${}^{0}_{-1}\mathrm{e} \Rightarrow \beta$, ${}^{1}_{0}\mathrm{n} \Rightarrow \mathrm{n}$, ${}^{1}_{0}\mathrm{n} \Rightarrow \mathrm{n}$
  - ${}^{1}_{1}\mathrm{H} \Rightarrow \mathrm{p}$, ${}^{2}_{1}\mathrm{H} \Rightarrow \mathrm{D/d}$, ${}^{3}_{1}\mathrm{H} \Rightarrow \mathrm{T/t}$
- Reaction notation: $A + B \rightarrow C + D$; Shorthand: $A(B, C)D$
  - e.g. $n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + p$ or ${}^{14}_{7}N(n, p){}^{14}_{6}C$

1.2.2. Fundamental laws governing nuclear reactions

- Conservation of nucleons
  - Total "A" remains the same
- Conservation of charge
  - Total "Z" remains the same
- Conservation of momentum
- Conservation of Energy
  - Energy, including rest mass, is conserved

1.2.3. Most important reactions

- Nuclear fission (n, f): ${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}X + \nu\mathrm{n} + \mathrm{E}$

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- Radiative capture (n, γ): ${}_0^1n + {}_Z^AX \rightarrow \left({}_Z^{A+1}X\right)^* \rightarrow {}_Z^{A+1}X + \gamma$
- Elastic scattering (n, n): ${}_0^1\mathrm{n} + {}_Z^AX \rightarrow {}_0^1n + {}_Z^AX$
- Inelastic scattering (n, n'): ${}_0^1n + {}_Z^AX \rightarrow {}_0^1n + \left({}_Z^AX\right)^* \rightarrow {}_0^1n + {}_Z^AX + \gamma$
- Alpha decay: ${}_Z^AX \rightarrow \alpha + {}_{Z-2}^{A-4}Y + E$
- Beta decay: ${}_Z^AX \rightarrow {}_{-1}^0e + {}_{Z+1}^AY + \bar{\nu} + E$

1.2.4. Energetics

- Nuclear reaction energetics governed by Einstein's equation
  - $E_{tot} = mc^2$
  - $m = \dfrac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$
    - $v \ll c \Rightarrow m = m_0\left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + O\left(\frac{v}{c}\right)^4\right]$

      $\Rightarrow E_{tot} \approx \underbrace{m_0c^2}_{\text{rest mass}} + \underbrace{\frac{1}{2}m_0v^2}_{KE:=E}$
  - For *neutrons* and *nuclides* in the reactor $v \ll c \Rightarrow E = \frac{1}{2}m_Xv^2$
  - For some high energy electrons $v \sim c \Rightarrow E \coloneqq E_{tot} - m_0c^2$
  - For gamma rays with no rest mass $E = h\nu$

Problem 1.5. [EEL]

The average kinetic energy of a fission neutron is 2.0 MeV. Defining the kinetic energy as $\mathrm{E} = \mathrm{E_{total}} - \mathrm{m_0c^2}$, what is the percent error introduced into the kinetic energy from using $\mathrm{E} = \frac{1}{2}\mathrm{m_0v^2}$.

Solution

- $E = E_{total} - m_0c^2 = \left[\dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right]m_0c^2$
- $E' = \frac{1}{2}m_0v^2$

  $\Rightarrow \dfrac{E'}{E} = \dfrac{1}{2}\left[\dfrac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right]^{-1}\left(\frac{v}{c}\right)^2$

  Assuming $v \ll c$: $\frac{v}{c} = \frac{1}{c}\sqrt{\frac{2\,E'}{m_0}} = \frac{1}{c}\sqrt{\frac{2\,(2\,MeV)}{m_n}} = 0.065$

  $\Rightarrow \dfrac{E'}{E} = 0.9968 \Rightarrow \Delta = 0.3\%$

- Conservation of total energy
  - $E_A + M_Ac^2 + E_B + M_Bc^2 = E_C + M_Cc^2 + E_D + M_Dc^2$

    For γ ray $E + Mc^2 \rightarrow h\nu$
- Q-value of the reaction
  - $Q = E_C + E_D - E_A - E_B = (M_A + M_B - M_C - M_D)c^2$
    - $Q > 0$ – Exothermic/ exoergic
      - Net loss of rest mass
      - Net increase of kinetic energy
    - $Q < 0$ – Endothermic/ endoergic
      - Net increase in rest mass
      - Net loss of kinetic energy

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- $Q > 0 \Rightarrow m_\Sigma \downarrow, E_\Sigma \uparrow$: applicable for *chemical reaction too*!
  - But changes in mass in chemical reaction are too small to measure.

Problem 1.6 [EEL]

Approximately what fraction of the reactants mass is converted to energy in the following reactions:

a) $n + {}^{235}U \rightarrow$ Fission, $E = 200\ MeV$

b) $C + O_2 \rightarrow CO_2, E = 4\ eV$

Solution

a) $\Delta_1 = \dfrac{200\frac{MeV}{c^2}}{(235+1)\ u} \approx 0.0009 \approx 0.1\%$

b) $\Delta_2 = \dfrac{4\frac{eV}{c^2}}{(12+2\times16)u} \approx 0.097 \times 10^{-9} \approx 10^{-8}\%$

1.3 The Curve of Binding Energy

- Mass defects and binding energies determine whether nuclear reactions release or absorb energy.
  - **Mass defect:** $\Delta = ZM_P + NM_N - M_X > 0\ \forall X$
  - **Binding energy**: $BE = \Delta c^2$
- **Binding energy per nucleon** measures **nuclear stability**, with larger values indicating greater stability.
  - $\dfrac{BE}{A} = \dfrac{\Delta c^2}{N+Z}$

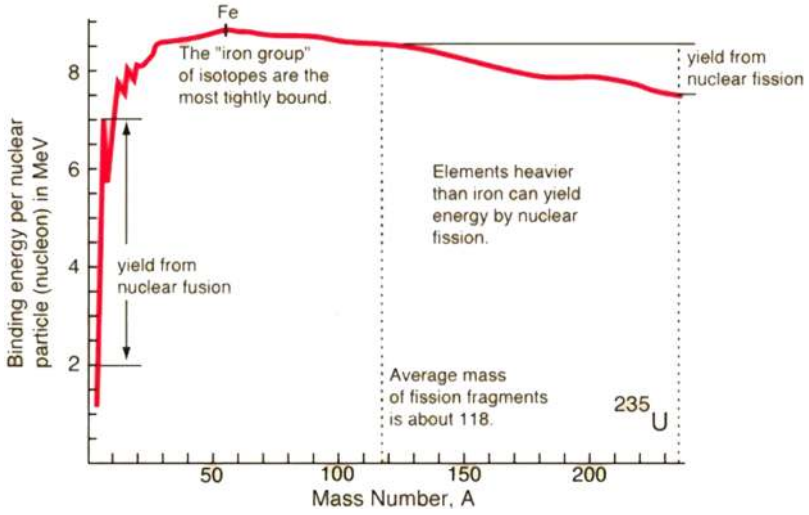

Fig. Curve of binding energy per nucleon

- **Exothermic reactions** produce more stable nuclei with increased binding energy, like **fusion** in the lower mass number region and **fission** in the higher mass number region.

1.4 Fusion Reactions

- Fusion involves charged particle reaction and requires high speed to overpower the coulomb repulsion
  - Two ways to impart high KE
    - Particle accelerator - uneconomical ($P_{prod} \ll P_{cons}$)
    - High temperature plasma (Difficulty of containment)
- Most widespread fusion reactions in research
  - $D - D$:
    - $D(D,n)\alpha + 3.25\ MeV$
    - $D(D,t)p + 4.02\ MeV$
    - Virtually unlimited fuel supply but difficult to initiate (requires higher T compared to

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$D-T$)

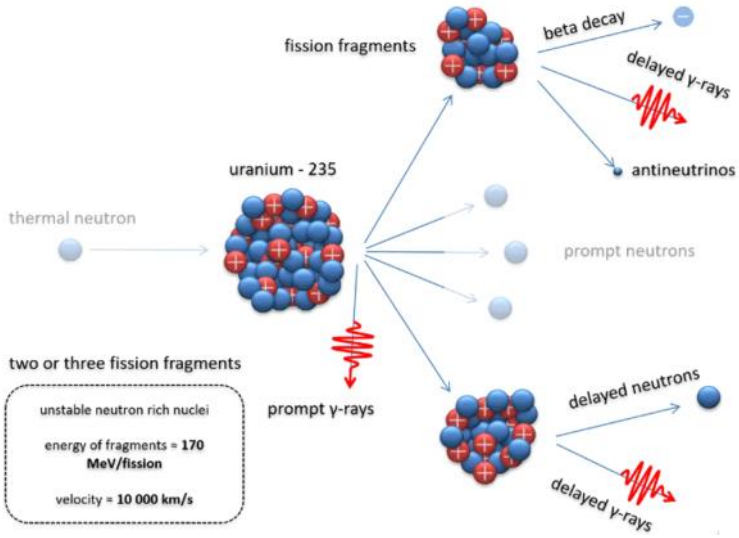
- $D-T$:
  - $T(D,n)\alpha + 17.6\ MeV$
  - Relatively easier to initiate but produces $14MeV$ neutrons which damage and activate wall materials

- Fission reactions, on the other hand, involve splitting heavy nuclei without requiring high temperatures and can be initiated by neutrons, making them more practical for controlled power production.

1.5 Fission Reactions

1.5.0. Introduction

- Fission of uranium-235 releases around 200 MeV of energy, two or three neutrons, fission fragments, gamma rays, and neutrinos.



- **Fission fragments** undergo radioactive decay, yielding more stable **fission products**.
- The energy, neutrons, and fission products are essential factors in the operation of nuclear power reactors, each with its own significant role.

1.5.1. Energy Release and Dissipation

- Fission reactions release energy as kinetic energy in fission fragments, neutrons, gamma rays, beta particles, and neutrinos.
- **Charged particles** (e.g., fission fragments, alpha, and beta particles) interact strongly with matter, rapidly losing energy and creating ion pairs that recombine, releasing heat.
  - Kinetic energy (KE) of fission fragments (FF) account for **80% of fission energy**
  - **Range of charged particles** is measured in **microns** ⇒ most of the fission energy deposited very close to the **point of fission.**
- **Neutral particles** (e.g. neutrons, gamma rays, and neutrinos) travel in straight lines and interact with matter differently.
  - Neutrons primarily scatter from nuclei before absorption, imparting energy to the recoiling nucleus striping its orbital electrons. [indirect ionization, range in cm]
  - Gamma rays scatter from electrons and nuclei. [indirectly ionizing, range in cm]
  - Neutrinos does not interact with matter and their energy is lost. [range infinite]
- The **energy dissipation mechanisms of charged and neutral particles** in biological tissues differ, with charged particles depositing energy over short distances and posing **inhalation or ingestion hazards**, while neutral particles primarily pose **external exposure hazards**.

1.5.2. Neutron Multiplication

- If fission neutrons are absorbed in a fissionable nuclide, it causes further fission and give birth to next generation neutrons and creating the possibility of **chain reaction.** *Chain reaction is characterized by* ***multiplication factor (k)*** defines the ratio of fission neutrons in one generation to the previous generation.
- Neutron population, given l − **neutron lifetime** [*birth by fission → scattering → absorption*]

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- $t = 0 \Rightarrow n = n_0$ – zeroth gen neutrons
- $t = l \Rightarrow n = kn_0$ – first gen neutrons
- $t = 2l \Rightarrow n = k^2 n_0$ – second gen neutrons
- $t = il \Rightarrow n = k^i n_0$ – i-th gen neutrons

$$i = \frac{t}{l} \Rightarrow n(t) = n_0 k^{\frac{t}{l}}$$

- Neutron population changes based on k: **supercritical** ($k > 1$), **subcritical** ($k < 1$), **critical** ($k = 1$).
- **Elementary equation of reactor kinetics**:
  - $k \sim 1 \Rightarrow ln(k) \approx k - 1 \Rightarrow n(t) = n_0\, exp\left[\left(\frac{t}{l}\right) ln(k)\right] \approx n_0\, exp\left[\frac{(k-1)t}{l}\right]$

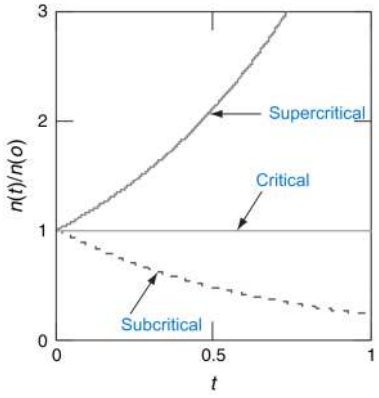
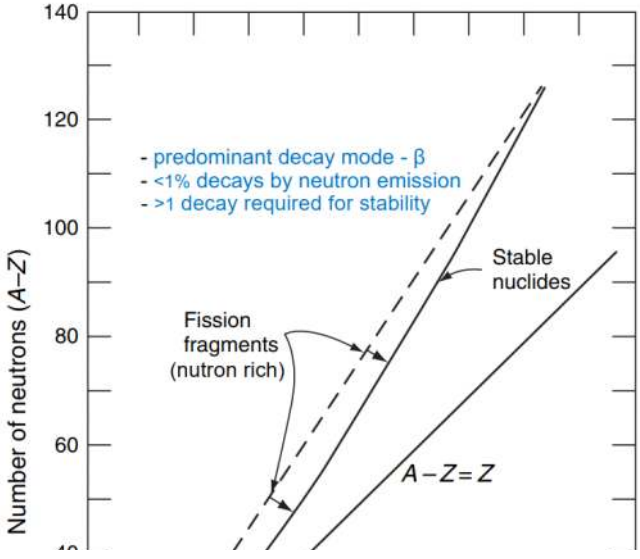


Fig.: Neutron population versus time

- Much of the reactor physics revolves around the multiplication coefficient k.
  - How k depends on the **composition** and **size** of the reactor.
  - Impact of **delayed neutrons**, **temperature change**, **fuel depletion** etc. on k.

1.5.3. Fission Products

- Fission usually yields pairs of fragments, with one heavier than the other, which are unstable due to high N/Z ratio, and decays by beta/ neutron emission to more stable fission products.
  - Example of fission → fission fragments → fission products
    - $n + {}^{235}_{92}U \rightarrow \underbrace{{}^{140}_{54}Xe + {}^{94}_{38}Sr}_{\text{fission fragments}} + \underbrace{2n}_{\text{prompt neutrons}} + 200MeV.$
      - ${}^{140}_{54}Xe \xrightarrow[t_{1/2}=13.6\ s]{\beta^-} {}^{140}_{55}Cs \xrightarrow[63.7\ s]{\beta^-} {}^{140}_{56}Ba \xrightarrow[12.8\ d]{\beta^-} {}^{140}_{57}La \xrightarrow[1.7\ d]{\beta^-} {}^{140}_{58}Ce.$ (unstable)
      - ${}^{94}_{38}Sr \xrightarrow[75.3\ s]{\beta^-} {}^{94}_{39}Y \xrightarrow[18.7\ min]{\beta^-} {}^{94}_{40}Zr.$
    - In the decay chains of FF, earlier half-lives are <u>usually</u> shorter than later ones.
  - 

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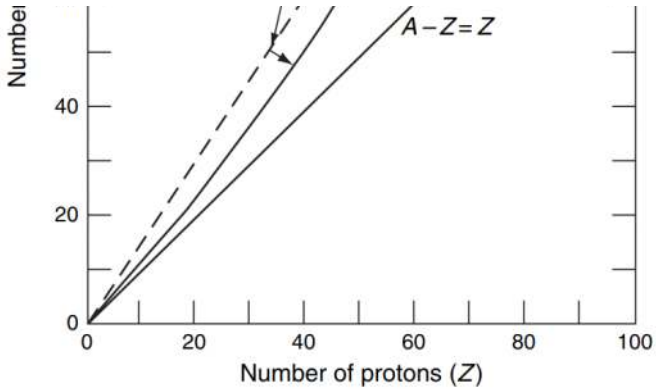


Fig.: Fission fragment instability

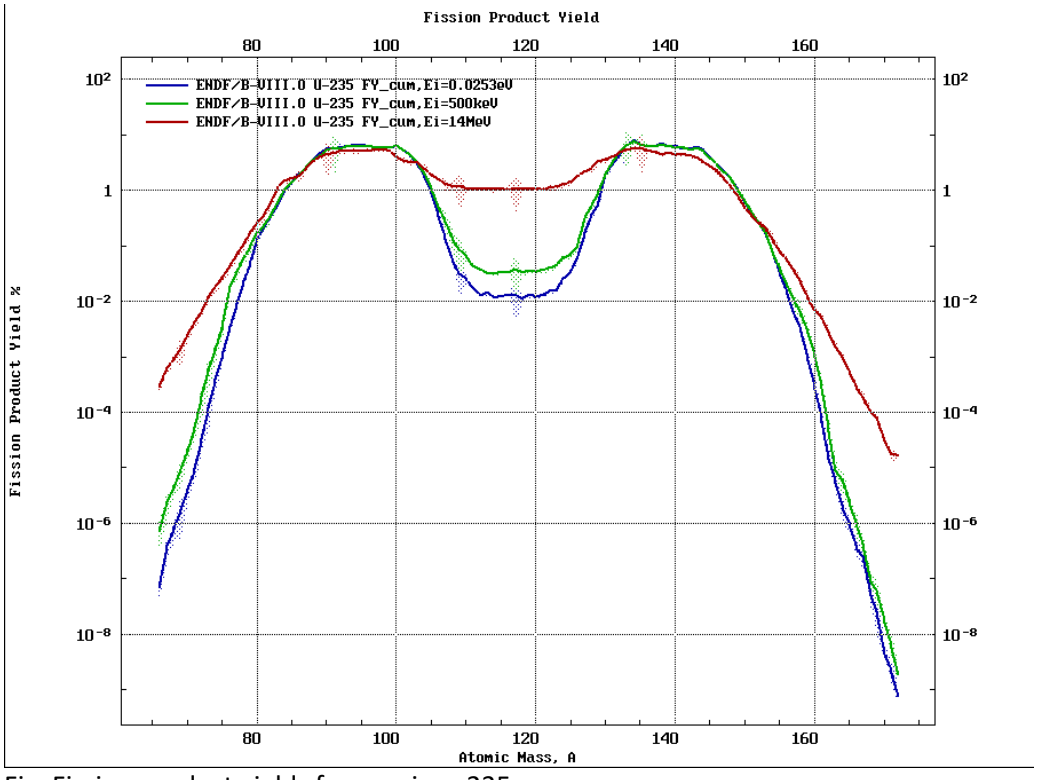
- **Fission Product Yields**
  - Fission produces over 40 different fission fragment pairs
    - Light group mass: $80 - 110$
    - Heavy group mass: $125 - 155$
  - Higher-energy neutrons increase the probability of equal-mass fission products.
  - Radioactive decay results in over 200 different fission products in nuclear reactors.
  - Fission product yields of fissionable materials follows similar trend as exemplified by U-235.
  - 

Fig. Fission product yields for uranium-235.

- **Decay heat** of fission products
  - About 8% of the energy produced in fission comes from β decay and associated γ rays which continues to generate decay heat even after reactor shut down and necessitates cooling to prevent overheating for an extended period.
  - The decay heat is approximated by the **Wigner-Way formula**
    - $P_d(t) = 0.0622\, P_o \left[t^{-0.2} - \left(t_o + t\right)^{-0.2}\right]$
      - $\mathrm{P_d}$ – power due to decay of FPs
      - P_0 – power before shutdown
      - $t_0\ [s]$ – period of power operation before shutdown
      - $t\ [s]$ – time elapsed since shutdown

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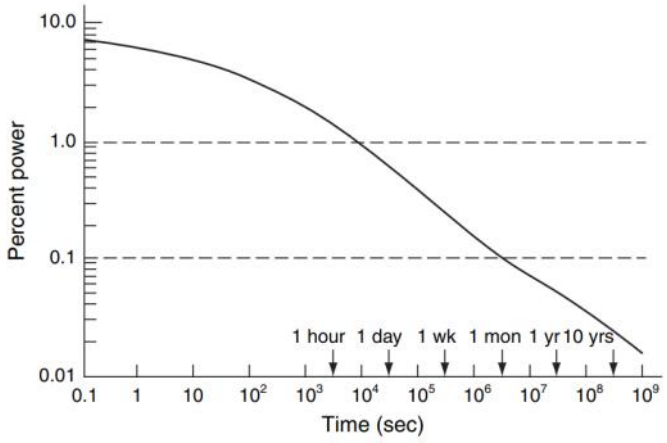

Fig. Heat produced by decay of fission products

1.5.4. Fissionable Materials

- **Fissile**: Undergoes fission when bombarded by neutrons of any energy (e.g. $^{235}_{92}U$, $^{233}_{92}U$, $^{239}_{94}Pu$)
- **Fertile**: Captures a neutron, then transforms via radioactive decay into a fissile material (e.g. $^{232}_{90}Th$, $^{238}_{92}U$, $^{240}_{94}Pu$)
  - $\mathrm{n} + \underbrace{{}^{238}_{92}\mathrm{U}}_{\substack{\text{fertile}\\ 99.3\%\,{}^{\mathrm{Nat}}\mathrm{U}}} \longrightarrow {}^{239}_{92}\mathrm{U} \xrightarrow{\beta} {}^{239}_{93}\mathrm{Np} \xrightarrow{\beta} \underbrace{{}^{239}_{94}\mathrm{Pu}}_{\substack{\text{fissile}\\ t_{1/2}=24.4\,\mathrm{k\,yrs}}}$
  - $\mathrm{n} + \underbrace{{}^{232}_{90}\mathrm{Th}}_{\substack{\text{fertile}\\ \text{abundant}}} \longrightarrow {}^{233}_{90}\mathrm{Th} \xrightarrow{\beta} {}^{233}_{91}\mathrm{Pa} \xrightarrow{\beta} \underbrace{{}^{233}_{92}\mathrm{U}}_{\text{fissile}}$
- **Breeding**
  - If more than two neutrons are produced per fission ($^{235}_{92}U \sim 2.4$), then one neutron can sustain chain reaction and more than one can be used to convert fertile to fissile material.
  - A reactor that creates more fissile material than it consumes is a **breeder reactor**.
  - Sustaining breeding is a major challenge due to unproductive neutron losses and most reactors use more fissile materials than they produce.
- **Shorthand notation** for fissionable materials
  - $^{232}_{90}Th \rightarrow 02;\ ^{235}_{92}U \rightarrow 25;\ ^{238}_{92}U \rightarrow 28;\ ^{239}_{94}Pu \rightarrow 49.$

1.5.5. Neutron Sources for Chain Reaction Initiation

- Some neutrons are naturally produced from high-energy cosmic rays colliding with nuclei which can trigger chain reaction.
- Widely used, reliable, and strong neutron source comes from (α, n) reaction.
  - $^{226}_{88}Ra \xrightarrow[t_{1/2}=1600\ yrs]{\alpha} {}^{222}_{86}Rn$
  - $^{9}_{4}Be \xrightarrow{(\alpha,n)} {}^{12}_{6}C$

1.6 Radioactive Decay

1.6.0. Necessity in Reactor Physics

- Behavior of fission products
- Rates of fertile-fissile conversion
- Fission poisons etc.

1.6.1. Basics of Radioactivity

- **Law**: The probability of decay of a radioactive nucleus does not depend on its **age**.
  - If ΔN nuclei decay out of N nuclei during the time interval Δt then probability of decay of a single nucleus is $\frac{\Delta N/N}{\Delta t}$. Experimental evidences suggest a constant characteristic value for this quantity for a particular nuclide in the limits [Shultis and Faw, 2016]

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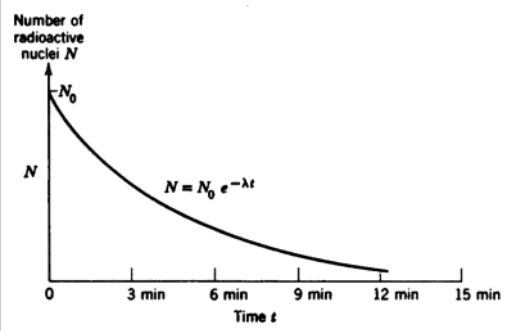
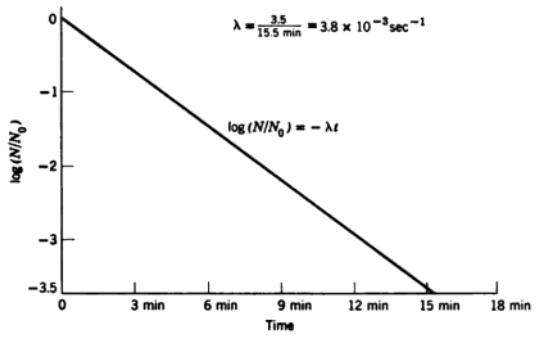
- $\lambda = \lim_{\Delta t \to 0} \frac{\Delta N/N}{\Delta t} = \frac{dN/N}{dt}$ = decay constant
- λ – decay probability of one radionuclide per unit time

- $(\lambda dt) \cdot N$ – expected number of decays in time dt of a sample with N radionuclides which must be equal to the decrease $-dN$ in numbers of the sample
  - $-dN = \lambda N dt \Rightarrow \frac{dN}{dt} = -\lambda N \Rightarrow N(t) = N(0) \cdot e^{-\lambda t}$
  - $\frac{dN}{dt} = -\lambda N$: number of nuclear decays or Activity is proportional to the toal number of undecayed nuclei. [$y' \propto y \rightarrow y \propto e^x \rightarrow$ exponential laws]





Radioactive decay of ^{218}Po

- **Half-life** ($t_{1/2}, T$)
  - $N\left(t_{1/2}\right) = N_0 e^{-\lambda t_{1/2}} = \frac{N_0}{2} \Rightarrow t_{1/2} = \frac{\ln(2)}{\lambda} \approx \frac{0.693}{\lambda}$
- **Mean-life** ($\bar{t}, \tau, T_A$)
  - $\bar{t} = \frac{\text{Total lifetime of all nuclei}}{\text{total number of nuclei}}$

    $\Rightarrow \bar{t} = \frac{\int_0^{N_0} t\, dN}{N_0} = \frac{1}{N_0}\int_\infty^0 t\, d(N_0 e^{-\lambda t}) = \int_\infty^0 t\, d\left(e^{-\lambda t}\right) = te^{-\lambda t}\Big|_\infty^0 - \int_\infty^0 e^{-\lambda t} dt$

    $= \frac{1}{\lambda}$
- Units of Measurement
  - 1 becquerel $(Bq) = 1\ decays/second$
  - 1 Curie $(Ci) = 3.7 \times 10^{10} Bq$ [Origin: Activity of 1g of ^{226}Ra]
    - $A_{Ra}^{1g} = \lambda_{Ra} \times N_{Ra}^{1g} = \frac{\ln 2}{t_{1/2}^{Ra}} \times \left(\frac{m_{Ra}}{M_{Ra}} N_A\right) = \frac{0.693}{1600\ yr} \times \frac{1}{226} \times 6.023 \times 10^{23}$

      $= 3.66 \times 10^{10}\ s^{-1}$

1.6.2. Saturation Activity

- Fission fragments are generated at a constant rate during *steady-power operation* and to determine the time dependence of their **inventory**, a **source term** A_0 is added to the decay equation:
  - $\frac{d}{dt} N(t) = A_0 - \lambda N(t)$
- Multiplying the ODE by **integrating factor** $e^{\lambda t}$
  - $e^{\lambda t} \cdot \frac{dN}{dt} = e^{\lambda t}\left[A_0 - \lambda N(t)\right]$
  - $\Rightarrow e^{\lambda t} \cdot \frac{dN(t)}{dt} + N(t) \cdot \lambda e^{\lambda t} = \frac{d}{dt}\left[N(t) e^{\lambda t}\right] = A_0 e^{\lambda t}$

    $\Rightarrow \int_0^t d\left[N(\tau) e^{\lambda \tau}\right] = \int_0^t A_0\, e^{\lambda \tau} d\tau \Rightarrow N(t) e^{\lambda t} = \frac{A_0}{\lambda}\left[e^{\lambda t} - 1\right] \leftarrow N(0) := 0$

    $\Rightarrow A(t) = \lambda N(t) = A_0\left(1 - e^{-\lambda t}\right)$
- Case study
  - $\lambda t \ll 1 \Rightarrow e^{-\lambda t} \approx 1 - \lambda t$ – linear initial increase

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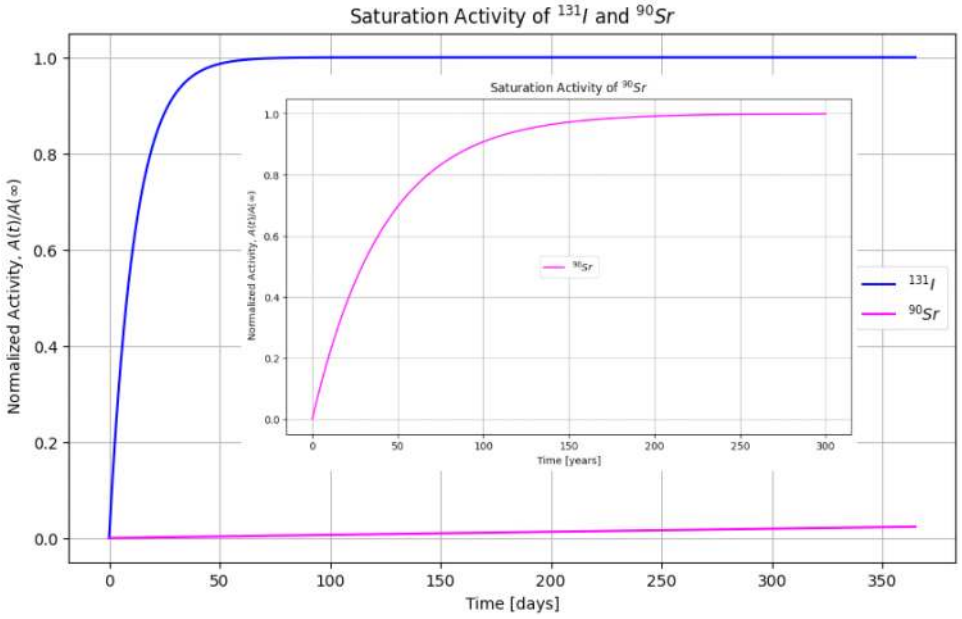
- $t \gg t_{1/2} \Rightarrow e^{-\lambda t} \to 0 \Rightarrow A(t=\infty) = \lambda N(\infty) = A_0$ – **saturation activity**.

- Illustration of saturation activity of Iodine-131 and Strontium-90
  - Initial data

$A_0^I = 0.85 \times 10^{18}$ nuclei/s	$t_{1/2}^I = 8.05\ d$	$\lambda_I = 0.086\ day^{-1}$
$A_0^{Sr} = 1.63 \times 10^{18}$ nuclei/s	$t_{1/2}^{Sr} = 10{,}628\ d$	$\lambda_{Sr} = 6.52 \times 10^{-5}\ day^{-1}$

  - Activity comparison
    - ^{131}I with shorter half-life reaches saturation in months
    - ^{90}Sr with longer half-life increases linearly even after years of reactor operation and reaches saturation only after 200 years!



1.6.3. Decay Chains

- Two stage decay equation for fission fragments
  - $X_1 \xrightarrow{\lambda_1} X_2 \xrightarrow{\lambda_2} X_3$
- Governing equation for X_1
  - $\frac{d}{dt}N_1(t) = -\lambda_1 N_1(t) \Rightarrow N_1(t) = N_0 e^{-\lambda_1 t}$
- Governing equation for X_2 [to solve by integrating factor]
  - $\frac{d}{dt}N_2(t) = \lambda_1 N_1(t) - \lambda_2 N_2(t) \Rightarrow e^{\lambda_2 t}\left[\frac{d}{dt}N_2(t)\right] = [\lambda_1 N_1(t) - \lambda_2 N_2(t)] \cdot e^{\lambda_2 t}$

    $\Rightarrow \frac{d}{dt}\left[N_2 e^{\lambda_2 t}\right] = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} \Rightarrow \int_0^t d\left[N_2(\tau)e^{\lambda_2 \tau}\right] = \lambda_1 N_0 \int_0^t e^{(\lambda_2 - \lambda_1)\tau} d\tau$

    $\Rightarrow N_2(t) \cdot e_2^{\lambda_2 t} - N_2(0) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left[e^{(\lambda_2 - \lambda_1)t} - 1\right]$

    $$\therefore N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right] \leftarrow N_2(0) := 0 \qquad (N2)$$

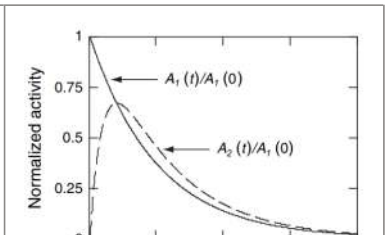
  - Case studies

• $\lambda_1 \ll \lambda_2\ (T_1 \gg T_2)$

$\Rightarrow N_2 \approx \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}\left[1 - e^{(\lambda_1 - \lambda_2)t}\right]$

$\Rightarrow N_2 \approx \frac{\lambda_1}{\lambda_2} N_1 \left[1 - e^{-\lambda_2 t}\right]$

$\Rightarrow \lambda_2 N_2 \approx \lambda_1 N_1\ \forall t \gg T_2$



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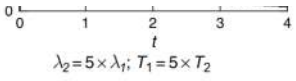
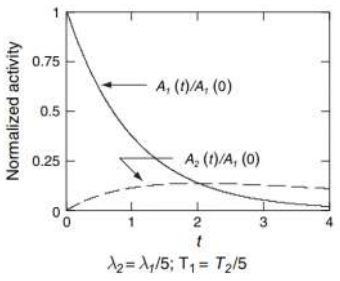
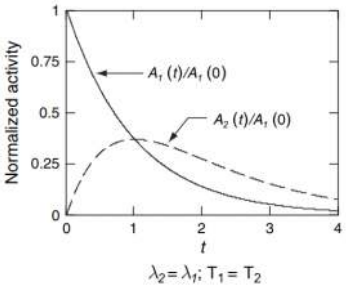
$\Rightarrow A_1 \approx A_2$ – secular equilibrium ($\frac{dN_2}{dt} = 0$)	
• $\lambda_1 \gg \lambda_2$ $(T_1 \ll T_2)$ $\Rightarrow N_2 \approx N_0 e^{-\lambda_2 t}\left[1 - e^{-(\lambda_1+\lambda_2)t \approx -\lambda_1 t}\right]$ $\Rightarrow N_2 \approx N_0 e^{-\lambda_2 t}\ \forall t \gg T_1$	
• $\lambda_1 \simeq \lambda_2$ $(T_1 \simeq T_2)$ ❖ $\lambda_1 = \lambda_2 \Rightarrow$ indeterminate $0/0$ ✧ $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$ ❖ $\lambda_1 \gtrsim \lambda_2$ – no equilibrium ❖ $\lambda_1 \lesssim \lambda_2$ ($T_1 \gtrsim T_2$) $\Rightarrow N_2 = \frac{\lambda_1 \cdot N_0 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}\left[1 - e^{(\lambda_1-\lambda_2)t}\right]$ $\Rightarrow N_2 \approx \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1\ \forall t \gg T_1$ $\Rightarrow \frac{N_2}{N_1} \approx const.$ – transient equilibrium	

- Governing equation for X_3
  - $\frac{d}{dt} N_3(t) = \lambda_2 N_2(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_0\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$
    $\Rightarrow N_3(t) = N_0 \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}\left[\frac{1}{\lambda_1}\left(1 - e^{-\lambda_1 t}\right) + \frac{1}{\lambda_2}\left(1 - e^{-\lambda_2 t}\right)\right] \leftarrow N_3(0) \coloneqq 0$
- **Multi-stage decay [Bateman equations]**

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2. Neutron interactions

Wednesday, September 27, 2023 7:57 AM

2.1. Introduction

- The behavior of the **fission neutrons** as they **interact with matter** determines the nature of neutron chain reactions. For a **sustained chain reaction**, *at least one of the fission neutrons must induce another fission*.
- This chapter describes **characterization of neutron interactions**, **energy range of neutrons**, and **energy dependence of interactions**.

2.2. Neutron Cross Sections

2.2.0. Introduction

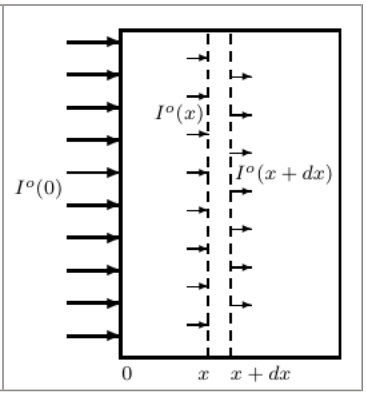
- Due to charge neutrality neutron's motion is unaffected by electromagnetic field of electrons or the nucleus and the motion can change only during collision with a nucleus.
- Thus neutrons travel in straight lines between collisions before ending their life in absorption.
- Neutrons perceive space in a solid mostly empty. [very small ratio of XS areas $\perp \overrightarrow{v_n}$]
  - $\frac{S_N}{S_A} = \frac{\pi R_N^2}{\pi R_A^2} \approx \frac{(10^{-12} cm)^2}{(10^{-8} cm)^2} = 10^{-8}.$

2.2.1. Macroscopic Cross Section

- **Definition:** Interaction of neutrons with matter is characterized by linear interaction coefficient/ Macroscopic cross section defined as the *probability of interaction per unit path length* which approaches a constant (empirically found) as the path length becomes infinitesimally small.
  - $\Sigma = \lim_{\Delta x \to 0} \frac{P(\Delta x)}{\Delta x}$
- **Neutron interaction in a slab**

□ Parallel beam of neutrons travelling in x direction with intensity of uncollided particles at depth x of the slab
  - $I^o(x)\ \left[\frac{\text{particles}}{cm^2\ s}\right]$

□ The probability an uncollided particle interacts as it crosses Δx
  - $P(\Delta x) = \frac{I^o(x) - I^o(x + \Delta x)}{I^o(x)}$



□ By the definition of Microscopic cross section
  - $\Sigma = \lim_{\Delta x \to 0} \frac{P(\Delta x)}{\Delta x}$
  - $\Rightarrow \lim_{\Delta x \to 0} \frac{I^o(x) - I^o(x + \Delta x)}{\Delta x} \frac{1}{I^o(x)} \equiv -\frac{dI^\circ(x)}{dx} \frac{1}{I^o(x)}$
  - $\Rightarrow \frac{dI^\circ(x)}{dx} = -\Sigma I^o(x)$
  - $\therefore \mathrm{I^\circ(x) = I^o(0)e^{-\Sigma x}}$

□ Probability of a neutron interaction in path length x
  - $P(x) = \frac{I^o(0) - I^o(x)}{I^o(0)} = 1 - e^{-\Sigma x}$
    - $x \to dx \Rightarrow P(\mathrm{d}x) = 1 - e^{-\Sigma \mathrm{d}x} \to 1 - (1 - \Sigma \mathrm{d}x) = \Sigma \mathrm{d}x$

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$$\Rightarrow \Sigma = \frac{dP}{dx} - as\ defined!$$

- Probability of a neutron does not interaction in path length x
  - $\bar{P}(x) = 1 - P(x) = e^{-\Sigma x}$

Mean Free Path

- The probability that neutron undergoes first interaction in the interval between x and $x + dx$
  - $p(x)dx = \{$prob. particle travels a distance x without interaction$\}\times\{$prob. it interacts in the next dx$\}$
    $\Rightarrow p(x)dx = \bar{P}(\mathrm{x}) \cdot \mathrm{P(dx)} = \left(e^{-\Sigma x}\right) \cdot (\Sigma \mathrm{d}x)$
    $\therefore p(x) = \Sigma e^{-\Sigma x} -$ probability distribution function (pdf)
- $p(x)$ is normalized to unity as required for a proper pdf
  - $\int_0^\infty p(x)dx = \int_0^\infty \Sigma e^{-\Sigma x}dx = \int_\infty^0 \mathrm{d}e^{-\Sigma x} = e^{-\Sigma x}\Big|_\infty^0 = 1$
- The average distance neutron travels before it interacts is called mean free path, λ, which can be found using the pdf $p(x)$
  - $\lambda = \int_0^\infty xp(x)dx = \int_0^\infty x\Sigma e^{-\Sigma x}dx = \int_\infty^0 x\, de^{-\Sigma x}$
    $\Rightarrow \lambda = xe^{-\Sigma x}\Big|_\infty^0 - \int_\infty^0 e^{-\Sigma x}dx = \frac{1}{\Sigma}$
- Thus Σ can also be interpreted as the inverse of the average distance traveled before interaction.
- Alternative "intuitive" derivation of λ [Liverhant-1960 4.7.]
  - Uncollided neutron beam intensity is proportional to the uncollided neutron density ($I^o = n^o v$)
    - $n^o(x) = n^o(0) \cdot e^{-\Sigma x} \Rightarrow dn^o = -n^o(0) \cdot \Sigma \cdot e^{-\Sigma x}dx$
  - Combined path length of dn^o neutrons that travelled uncollided upto x, undergo collision in between x and $x + dx$ is thus $x\ dn^o$.
    - Total combined path length $= \int_{n^o(0)}^0 x\ dn^o$
  - Average path length
    - $\lambda = \frac{\int_{n^o(0)}^0 x\ dn^o}{n^o(0)} = \frac{-\int_0^\infty x \cdot n^o\ (0) \cdot \Sigma \cdot e^{-\Sigma x}dx}{n^o(0)} = \int_\infty^0 x de^{-\Sigma x} = \frac{1}{\Sigma}$

Reaction rate

- If a neutron moving at velocity v, has a mean free path λ, then mean time τ between successive collisions for the given neutron:
  - $\tau = \frac{\lambda}{\mathrm{v}} \quad \Rightarrow \nu = \frac{1}{\tau} = \frac{\mathrm{v}}{\lambda} -$ **interaction frequency**
- If there are n neutrons per cm^3 then interaction rate per cm^3 (or reaction rate density) will be
  - $r = \frac{\mathrm{v}}{\lambda} \cdot n = \Sigma \cdot (\mathrm{v} \cdot n) = \Sigma\phi$
  - $\phi = n\mathrm{v} =$ neutron flux density (or simply called neutron flux)
- Then reaction rate for a medium of volume V [cm^3]
  - $R = rV = \Sigma\phi V$

Uncollided Flux

- The uncollided neutron flux, given a density of uncollided neutrons (n^o) and their velocity (v), can be expressed as
  - $\phi^o(x) = \mathrm{v}n^o(x)$
- For a point isotropic neutron source in vacuum emitting S_p neutrons/sec radially outward uncollided flux at a distance r from the source:
  - $\phi^o(r) = \frac{S_p}{4\pi r^2}$ [geometric attenuation]

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- With a material present, only a fraction $exp(-\Sigma r)$ of the neutrons will survive to a distance r without undergoing a collision. Thus accounting for both **geometrical and material attenuations** the uncollided flux at a distance r from a point source:
  - $\phi^o(r) = \frac{e^{-\Sigma r}}{4\pi r^2} S_p$

2.2.2. Microscopic Cross Section

- Macroscopic cross section (Σ) depends on neutron energy (E), interaction type, and the composition/density of the medium. It seems reasonable to expect that Σ should be proportional to the "target" atom density N in the material.
  - $\Sigma \propto N \Rightarrow \Sigma = \sigma N$
- The proportionality constant σ is called the **microscopic cross section** and defined as *the interaction probability per unit differential path length, normalized to one target atom per unit volume.*
  - This definition avoids the philosophical difficulty of understanding how σ changes with neutron energy when defined as the effective target atom cross-section.
- Units of microscopic cross section: $cm^2/nuclide$, barn (b)
  - $1b = 10^{-24}\ cm^2$
  - Barn originated from early determinations of neutron cross sections when one of the investigators exclaimed, "That's as big as a barn" in reaction to one of the measurements.
- Microscopic cross section depends on neutron energy (E), type of interaction, and the type of nuclide.

2.2.3. Cross Section Calculation

Nuclide Densities

- Determination of macroscopic cross-section Σ requires determination of nuclide density (N). Calculaiton involving N relies on the concept of **moles** and that one mole of substance contain $N_0 = 0.6023 \cdot 10^{-24}$ (Avogadro) number of species.
  - Mass density: $\rho - [g/cm^3]$
    - Isotopic fraction, f_i
      - $f_i = \frac{N_i}{N} \Rightarrow N = \Sigma_i N_i \Rightarrow A = \sum_i f_i A_i$
      - $\Sigma = \sum_i N_i \sigma^i = N \sum_i \frac{N_i}{N} \sigma^i = \frac{\rho N_0}{A} \sum_i f_i\, \sigma^i$
  - Molar weight: $A - [g/mol]$
  - Molar density: $\rho/A - [mol/cm^3]$
  - Nuclide density: $N = \frac{\rho}{A} \cdot N_A\ [nuclide/cm^3]$

  $\Rightarrow \Sigma = \sigma N = \frac{\rho N_0}{A} \cdot \sigma$
- Molecular case
  - $\Sigma^{H_2O} = \frac{\rho_{H_2O} N_0}{18}\left(2\sigma^H + \sigma^O\right) = N_{H_2O}\sigma^{H_2O}\ \left[\Leftarrow \sigma^{H_2O} \overset{def}{=} 2\sigma^H + \sigma^O\right]$
- Mixture - Volume fraction, β_i
  - $\beta_i = \frac{V_i}{V} \Rightarrow V = \sum_i V_i\ \left[\text{Assuming, } \Delta V = 0 \text{ from mixing}\right]$
  - $\Sigma = \sum_i \beta_i \Sigma^i = \sum_i \beta_i\, N_i \sigma^i = \sum_i \beta_i \frac{\rho_i N_0}{A_i} \sigma^i$
- Mixture - Mass fraction, w_i
  - $\Sigma = \sum_i \beta_i \frac{\rho_i N_0}{A_i} \sigma^i$

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$$\Rightarrow \Sigma = \sum_i \frac{V_i}{V} \cdot \frac{\rho_i N_0}{A_i} \sigma^i = \sum_i \frac{\rho_i V_i}{\rho V} \cdot \frac{\rho N_0}{A_i} \sigma^i = \rho N_A \sum_i \frac{\mathrm{m_i}}{\mathrm{m}} \cdot \frac{1}{A_i} \sigma^i$$

Enriched Uranium

- Natural uranium
  - 0.7% uranium-235
  - 99.3% uranium-238
- Enriched uranium: increased the fissile-to-fertile ratio
- Enrichment by atom percent (a/o) [preferred]
  - $\tilde{e}_a = \frac{N^{25}}{N^{25}+N^{28}} \Rightarrow 1-\tilde{e}_a = \frac{N^{28}}{N^{25}+N^{28}}$
  - $\Sigma^U = \frac{\rho_U N_0}{\tilde{e}_a 235 + (1-\tilde{e}_a)238}\left[\tilde{e}_a\sigma^{25} + (1-\tilde{e}_a)\sigma^{28}\right] \left[\Leftarrow f_i\right]$
- Enrichment by weight percent (w/o) [practical]
  - $\tilde{e}_W = \frac{m^{25}}{m^{25}+m^{28}} \Rightarrow 1-\tilde{e}_W = \frac{m^{28}}{m^{25}+m^{28}}$
  - $\Sigma^U = \rho_U N_A \left[\frac{1}{235}\tilde{e}_W\sigma^{25} + \frac{1}{238}(1-\tilde{e}_W)\sigma^{28}\right] \left[\Leftarrow w_i\right]$
- Relation between a/o and w/o [based on moles]
  - $\tilde{e}_a = \frac{N^{25}/\mathrm{N_0}}{N^{25}/\mathrm{N_0}+N^{28}/\mathrm{N_0}} = \frac{\mathrm{mol}^{25}}{\mathrm{mol}^{25}+\mathrm{mol}^{28}} = \left(1+\frac{\mathrm{mol}^{28}}{\mathrm{mol}^{25}}\right)^{-1}$

    $$\Rightarrow \tilde{e}_a = \left(1+\frac{m^{28}/\mathrm{A}^{28}}{m^{25}/\mathrm{A}^{25}}\right)^{-1} = \left(1+\frac{\mathrm{A}^{25}}{A^{28}}\cdot\frac{\frac{m^{28}}{m_U}}{\frac{m^{25}}{m_U}}\right)^{-1}$$

    $$\Rightarrow \tilde{e}_a = \left(1+\frac{\mathrm{A}^{25}}{A^{28}}\cdot\frac{1-\tilde{e}_W}{\tilde{e}_W}\right)^{-1} = \left(\frac{\mathrm{A}^{28}}{A^{25}}\tilde{e}_W + 1 - \tilde{e}_W\right)^{-1} \cdot \frac{\mathrm{A}^{28}}{A^{25}}\tilde{e}_W$$

    $$\Rightarrow \tilde{e}_a = \left(1+\frac{\mathrm{A}^{28}-A^{25}}{A^{25}}\tilde{e}_W\right)^{-1} \cdot \frac{\mathrm{A}^{28}}{A^{25}}\tilde{e}_W = \left(1+0.0128\tilde{e}_W\right)^{-1} 1.0128\tilde{e}_W$$

  - $\tilde{e}_W = 0.007 \Rightarrow \tilde{e}_a = 0.00709.$
- Except for very precise calculation
  - $\Sigma^U \simeq \frac{\rho_U N_0}{238}\sigma^U \left[\Leftarrow \sigma^U \overset{\mathrm{def}}{=} \tilde{e}\sigma^{25} + (1-\tilde{e})\sigma^{28};\ \tilde{e} = \tilde{e}_a\right]$

Calculation example

- Calculate the cross sections for 8% enriched uranium dioxide $(\mathrm{UO_2})$ that is mixed in a 1: 3 volume ratio with graphite (C).
  - Given data
    - $\sigma^{25} = 607.5\ \mathrm{b}, \sigma^{28} = 11.8\ \mathrm{b}, \sigma^{\mathrm{O}} = 3.5\ \mathrm{b}, \sigma^{\mathrm{C}} = 4.9\ \mathrm{b}$
    - $\rho_{\mathrm{UO_2}} = 11.0\ \mathrm{g/cm^3}, \rho_{\mathrm{C}} = 1.60\ \mathrm{g/cm^3}$
  - microscopic cross section calculations
    - $\sigma^{\mathrm{U}} = 0.08 \cdot 607.5 + (1-0.08)\cdot 11.8 = 59.5\ \mathrm{b}.$
    - $\sigma^{\mathrm{UO_2}} = 59.5 + 2\cdot 3.5 = 66.5\ \mathrm{b}.$
  - Macro XS calculation
    - $\Sigma^{\mathrm{UO_2}} = \frac{11\cdot 0.6023\cdot 10^{24}}{238+2\cdot 16} 66.5\cdot 10^{-24} = 1.63\ \mathrm{cm^{-1}}.$
    - $\Sigma^{\mathrm{C}} = \frac{1.6\cdot 0.6023\cdot 10^{24}}{12} 4.9\cdot 10^{-24} = 0.39\ \mathrm{cm^{-1}}.$
    - $\Sigma = \frac{1}{4}\Sigma^{\mathrm{UO_2}} + \frac{3}{4}\Sigma^{\mathrm{C}} = \frac{1}{4}1.63 + \frac{3}{4}0.39 = 0.70\ \mathrm{cm^{-1}}.$

2.2.4. Reaction Types

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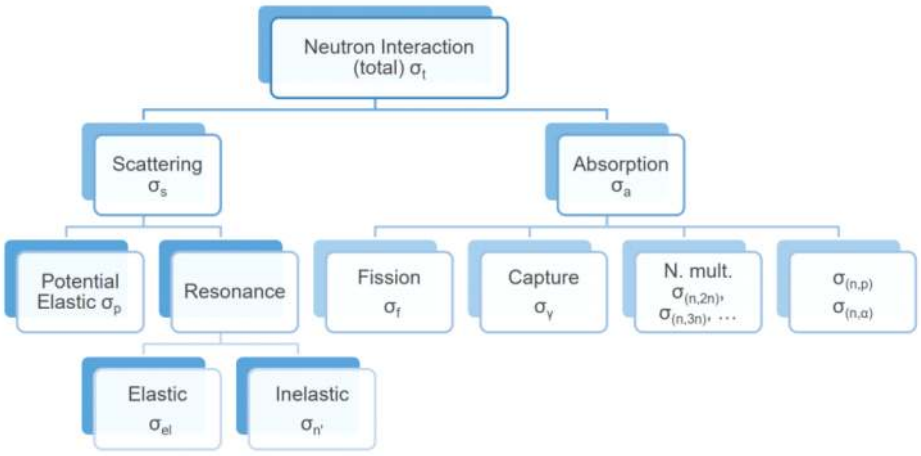
- So far we've looked at neutron interaction probabilities without detailing the specific reaction outcomes, which can be categorized as follows:



- Relations between different micro-cross sections:
  - $\sigma_t = \sigma_s + \sigma_a$
    - $\frac{\sigma_s}{\sigma_t}$ —probability of scattering
    - $\frac{\sigma_a}{\sigma_t}$ —probability of absorption
  - $\sigma_s = \sigma_n + \sigma_{n'}$
    - $\sigma_n = \sigma_p + \sigma_{el}$ = total elastic scattering cross section [$KE, \vec{p}$ = conserved]
    - σ_p — probability of scattering from the nuclear potential region w/o actually entering the nucleus [Hard-sphere collision]
    - σ_{el} — probability that neutron is quickly reemitted after entering the nucleus without invoking nuclear processes, conserving $KE, \vec{p}$
    - $\sigma_{n'}$ — the neutron gives up some of its energy to the nucleus, leaving it in an excited state.
  - $\sigma_a = \sigma_\gamma + \sigma_f$ — for fissionable materials
    - $\frac{\sigma_\gamma}{\sigma_a}$ — relative probability of radiative capture
    - $\frac{\sigma_f}{\sigma_a}$ —relative probability of fission
- Macroscopic cross section is also categorized analogously
  - $\Sigma_X = N\sigma_X$ where $x = s, a, \gamma, f$ etc.
  - $\Sigma_t = N\sigma_t = N(\sigma_s + \sigma_a) = N\sigma_s + N\sigma_a = \Sigma_s + \Sigma_a$.

2.3. Neutron Energy Range

- As noted earlier, cross sections vary with the kinetic energy of neutrons and this energy dependence is crucial for understanding chain reactions.
  - $\sigma_X = \sigma_X(E)$ and $\Sigma_X = \Sigma_X(E)$ — functions of energy
- Thus far we have considered monoenergetic neutrons (single velocity, v), but in the reactor neutrons have a spectrum of energy distributions.
- **Energy spectrum of fission neutrons**
  - **Watt distribution**: $\chi(E) = 0.453 \exp(-1.036E) \sinh\left(\sqrt{2.29E}\right)\ [\mathrm{MeV}^{-1}]$
  - Normalization: $\int_0^\infty \chi(E)dE = 1$
  - Average energy: $\overline{E} = \int_0^\infty E \cdot \chi(E)dE \approx 2\ MeV$
  - Most probable energy: $E_p \approx \frac{3}{4} MeV\ [\chi(E_p) = \max \chi]$
  - Reasonable maximum energy limit for fission neutrons, $E_{\max} \coloneqq 10\ \mathrm{MeV}$
    - $\int_0^{E_{\max}} \chi(E)dE \approx \int_0^\infty \chi(E)dE \Leftrightarrow \int_{E_{\max}}^\infty \chi(E)dE \approx 0$
- Neutrons lose energy in scattering collisions, transferring it to stationary nuclei, which have much

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lower energies ($\bar{E} = \frac{3}{2}kT \approx 0.0379eV@\,T = 293K$) compared to fission neutrons ($\bar{E} = 2\ MeV$) until they are absorbed.

- In the absence of absorption, neutrons would reach thermal equilibrium with the surrounding nuclei's motion, and take the famous **Maxwell-Boltzmann distribution**
  - $M(E) = \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} E^{\frac{1}{2}} \exp\left(-\frac{E}{kT}\right) \quad [\mathrm{eV}^{-1}]$
  - □ Normalization: $\int_0^\infty M(E)dE = 1$
  - □ Average energy: $\int_0^\infty E \cdot M(E)dE = \frac{3}{2}kT$
  - □ Most probable energy: $E_p = \frac{1}{2}kT \left[\Leftarrow \left.\frac{\mathrm{dM}}{\mathrm{dE}}\right|_{\mathrm{E_p}} = 0\right]$
  - □ Lower bound of energy range: $E_{\min} \coloneqq 0.001\ eV$

    $$\int_{\mathrm{E_{min}}}^{\infty} \mathrm{M}(E)dE \approx \int_0^{\infty} \mathrm{M}(E)dE \Leftrightarrow \int_0^{\mathrm{E_{min}}} \mathrm{M}(E)dE \approx 0$$
  - □ Absorption shifts spectrum up from M(E), preventing full equilibrium.

Problem [EEL 2.6]

Neutrons are distributed in the Maxwell-Boltzmann distribution given by $\mathrm{M(E)} = \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} E^{\frac{1}{2}} exp\left(-\frac{E}{kT}\right)$

a. Verify that $\int_0^\infty \mathrm{M(E)dE} = 1$
b. Verify that $\overline{\mathrm{E}} = \frac{3}{2}\mathrm{kT}$
c. Determine the most probably neutron energy, $\mathrm{E_p}$.

Solution

Look up Gaussian integral and double factorial!!

$\int_0^\infty x^{2n} e^{-bx^2} dx = \frac{(2n-1)!!}{b^n 2^{n+1}} \sqrt{\frac{\pi}{b}}$	$(2n-1)!! = 1 \cdot 3 \cdot 5 \cdots (2n-1)$

a. Proving $\int_0^\infty M(E)dE = 1$

$$\int_0^\infty M(E)dE = \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} \int_0^\infty E^{\frac{1}{2}} exp\left(-\frac{E}{kT}\right) dE$$

◇ $x^2 = \frac{E}{kT} \Rightarrow E = kTx^2 \Rightarrow dE = 2kTxdx \ \ and \ \sqrt{E} = \sqrt{kT}x$

$$\Rightarrow \int_0^\infty M(E)dE = \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} \cdot 2(kT)^{\frac{3}{2}} \int_0^\infty x^2\, exp(-x^2)dx = \frac{4}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{4} = 1$$

b. By definition

$$\bar{E} = \int_0^\infty EM(E)dE = \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} \int_0^\infty E^{\frac{3}{2}} exp\left(-\frac{E}{kT}\right) dE$$

◇ $x^2 = \frac{E}{kT} \Rightarrow E = kTx^2 \Rightarrow dE = 2kTxdx \ \ and \ E^{\frac{3}{2}} = (kT)^{\frac{3}{2}} \cdot x^3$

$$\Rightarrow \bar{E} = \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} \cdot 2(kT)^{\frac{5}{2}} \int_0^\infty x^4\, exp(-x^2)dx = \frac{4kT}{\sqrt{\pi}} \cdot \frac{3}{8}\sqrt{\pi} = \frac{3}{2}kT$$

c. Most probable value E_p occurs when $M(E)$ reaches maximum.

$$\left.\frac{dM(E)}{dE}\right|_{E_p} = 0 \Rightarrow \frac{2\pi}{(\pi kT)^{\frac{3}{2}}} \left.\frac{d}{dE} E^{\frac{1}{2}} exp\left(-\frac{E}{kT}\right)\right|_{E_p} = 0$$

$$\Rightarrow \left[\frac{1}{2\sqrt{E_p}} - \frac{\sqrt{E_p}}{kT}\right] \cdot exp\left(-\frac{E_p}{kT}\right) = 0 \Rightarrow E_p = \frac{1}{2}kT$$

- Classification Based on Neutron Energies

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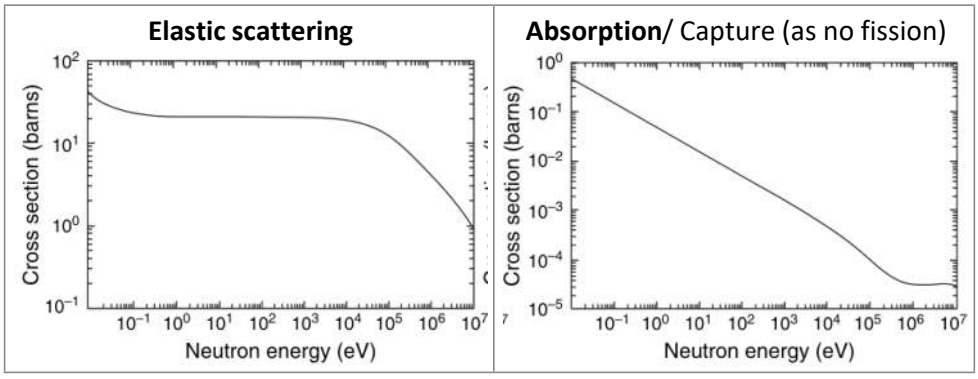
- Fast Neutrons: $0.1MeV < E < 10MeV$.
- Epithermal (Intermediate Energy) Neutrons: $1.0eV < E < 0.1MeV$.
- Thermal Neutrons: $0.001eV < E < 1.0eV$.

2.4. Cross Section Energy Dependence

2.4.0. Introduction

- *Cross section energy dependence of Hydrogen*
  - Hydrogen (^1H) consists of a single proton and has only two cross sections



  - *No inelastic scattering in hydrogen due to lack of internal structure.*
  - Except for the high and low energy tails, elastic potential scattering is energy independent.
  - Capture cross section, known as "one-over-v"
    - $\sigma_a(E) \propto \frac{1}{\sqrt{E}} \sim \frac{1}{v} \Rightarrow \sigma_a(E) = \sqrt{\frac{E_0}{E}} \cdot \sigma_a(E_0)$
    - Conventionally, $E_0 = kT \coloneqq 0.0253\ eV\ @T = 293K$
  - ^2H or deuterium cross sections
    - Similar behavior as hydrogen.
    - Larger scattering cross section.
    - Smaller absorption cross section.
- *Overview of potential scattering cross sections*
  - Analogous to billiard ball collisions - kinetic energy is conserved.
  - Neutron scatters from the surface of the nucleus, rather than entering its interior to form a compound nucleus.
  - ***Energy independent except at extremities.***
  - Proportional to nucleus's cross-sectional area with radius: $\mathrm{R} = 1.25 \times 10^{-13} \cdot \mathrm{A}^{1/3}\ \mathrm{cm}$

2.4.1. Compound Nucleus Formation

- *Excitation of Compound Nucleus*
  - When neutron enters a nucleus, compound nucleus is formed, which is in excited state due to
    - Kinetic energy of the neutron.
    - Binding energy of the neutron (E_B)
  - During formation of compound nucleus, momentum is conserved but kinetic energy might not.
    - $mv = (m + Am)\mathrm{V}$
    - $\Delta E_{ke} = \frac{1}{2}mv^2 - \frac{1}{2}(m + Am)\mathrm{V}^2 = \frac{1}{2}mv^2\left(1 - \frac{m}{m + Am}\right)$
      $\Rightarrow \Delta E_{\mathrm{ke}} = \frac{A}{1 + A}\frac{1}{2}mv^2 = E_{cm}$
      $E_{cm} -$ neutron KE in the center of mass system
  - Slow moving thermal neutrons, too, can excite a nucleus because even though $E_{cm} \ll E_B$, the binding energy may amount to MeV or more.

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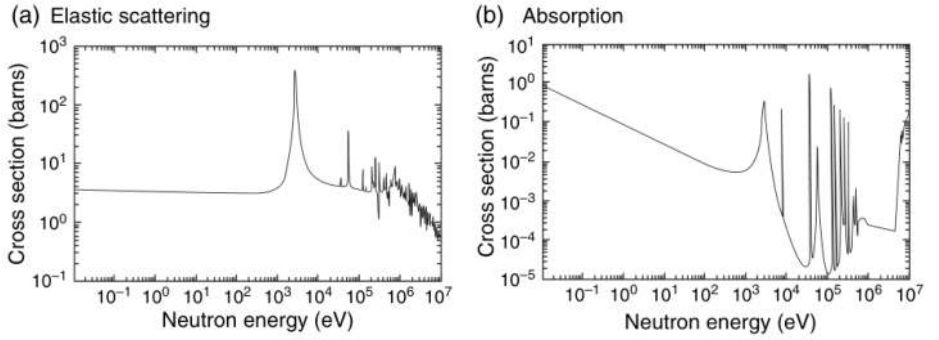
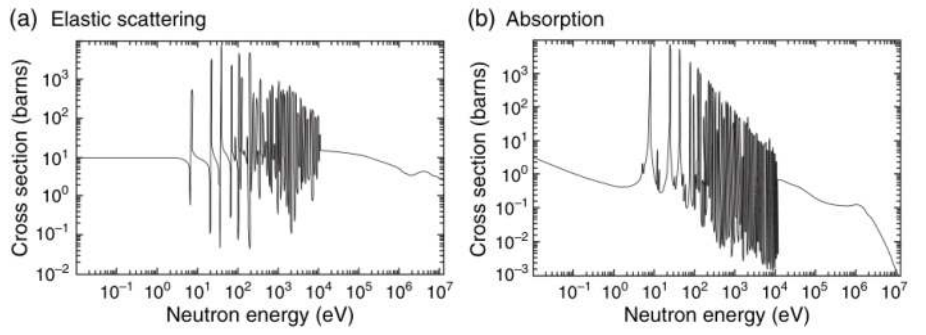
- *Outcomes of Compound Nucleus*
  - Neutron can be re-emitted (**elastic scattering**).
  - **Neutron capture** with gamma ray emission, creating a new isotope.
  - At higher neutron energies **inelastic scattering** results in neutron and gamma ray emission.
  - **Fission reactions** are crucial in fissile and fertile materials.
  - Other reactions $(n, xn), (n, \alpha)$ etc. might also occur at high energies.

2.4.2. Resonance Cross Sections

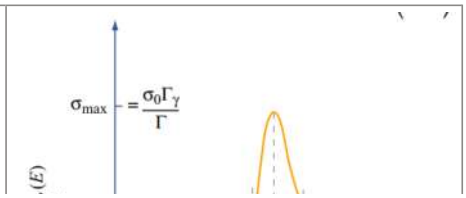
- *Internal nuclear structure and the resonance cross sections*
  - Like electrons in the atom, nucleons in the nucleus have quantum energy states.
  - Compound nucleus formation is most likely when the incident neutron's energy matches a quantum state of the resulting nucleus.
  - Scattering and absorption cross sections exhibit resonance peaks at neutron kinetic energies corresponding to those quantum states.

Micro cross sections of ^{23}Na	
Micro cross sections of ^{238}U	

  - Every nuclide has its distinct resonance structure, but generally **heavy nuclei** tend to have **more energy states** which are **densely packed**. In ^{238}U resonances seem to cease around 10keV, but in reality, they persist but are too densely packed to resolve.
  - Resonances for lighter nuclei begin to occur at higher energies (^{12}C: 2 MeV, ^{16}O: 400 keV, ^{23}Na: 3 keV, ^{238}U: 6.6 eV) and are more widely spaced, with a smaller capture-to-scattering cross-section ratio.

- *Breit-Wigner Formula*
  - Predicts energy dependence of cross sections near each resonance.
  - Capture cross section formula:

$$\sigma_\gamma(E) = \sigma_O \frac{\Gamma_\gamma}{\Gamma}\left(\frac{E_r}{E}\right)^{\frac{1}{2}} \frac{1}{1 + 4\left(E - E_r\right)^2/\Gamma^2}$$

• Γ = Total line width (FWHM - Full Width at Half Maximum) • Γ_n = Radiative line width • σ_0 = total cross section at $E = E_r$	

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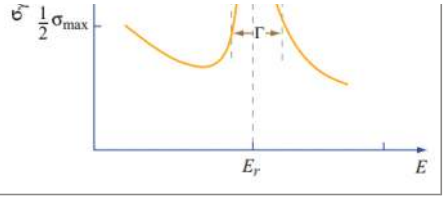
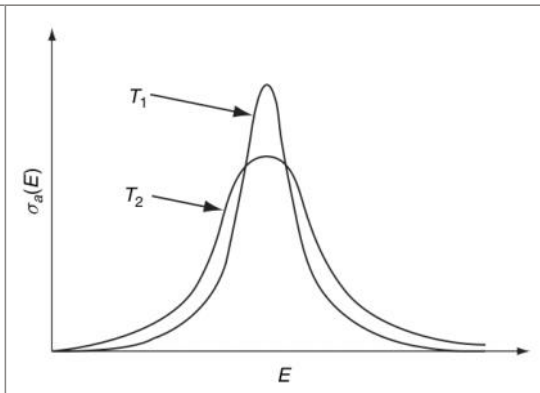
- Level width measures uncertainty in energy
  - $\Delta E = \frac{h}{2\pi\Delta t} = \frac{h}{2\pi\tau} = \frac{h}{2\pi}\lambda = \Gamma$
  - τ – mean decay time of the excited compound nucleus



- Elastic scattering formula:
  - $\sigma_n(E) = \sigma_o \frac{\Gamma_n}{\Gamma} \frac{1}{1 + 4(E - E_r)^2/\Gamma^2} + \sigma_o \frac{2R}{\lambda_o} \frac{2(E - E_r)/\Gamma}{1 + 4(E - E_r)^2/\Gamma^2} + 4\pi R^2$
  - 1st term - resonance scattering, 2nd term - QM interference between resonance and potential scattering (→ dip in scattering xs in heavy nuclides), 3rd term - potential scattering
- Elastic scattering cross section combines elastic resonance and elastic potential scattering
  - $\sigma_n(E) = \sigma_{nr}(E) + \sigma_{np}$

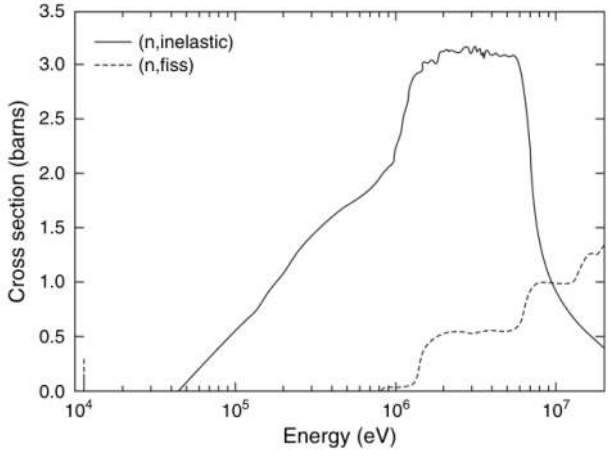
Doppler Broadening of Resonance Peaks

- Doppler broadening is a phenomenon where resonances in nuclear cross sections are spread out due to the thermal motions of target nuclei, analogous to the spread in observed frequencies (or energies) due to relative motion.

○ Thermal motion leads to some nuclei moving toward the incoming neutron and others moving away, resulting in a range of energies meeting resonance condition thus broadening the peak. ○ Broadening is more pronounced at higher temperatures, as illustrated in the figure for resonance capture cross-section at $T_1 < T_2$. ○ Later, we will discuss how Doppler broadening plays a crucial role in providing **negative temperature feedback and stability** to nuclear reactors.	

2.4.3. Threshold Cross Sections

- Higher neutron energies (→ higher excitation) enable "threshold reactions," which have zero cross-section below their specific **threshold energy**.
- Inelastic scattering shows threshold behavior, with lower threshold energy for heavier nuclei, in general. $^{12}C: 4.8\ MeV$, $^{238}U: 0.04\ MeV$.
- Fertile materials like uranium-238 have fission thresholds, with uranium-238 requiring approximately 1.0 MeV.



- Threshold behavior is exhibited in particle-emitting reactions like (n, 2n), but their higher

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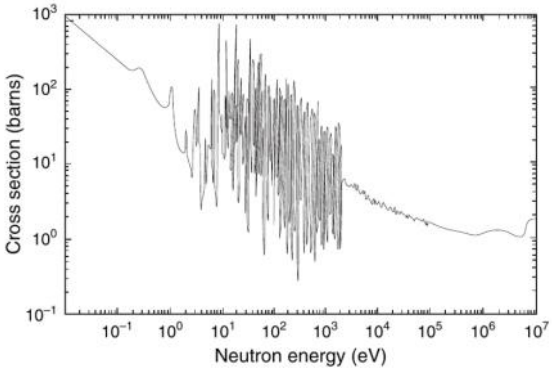
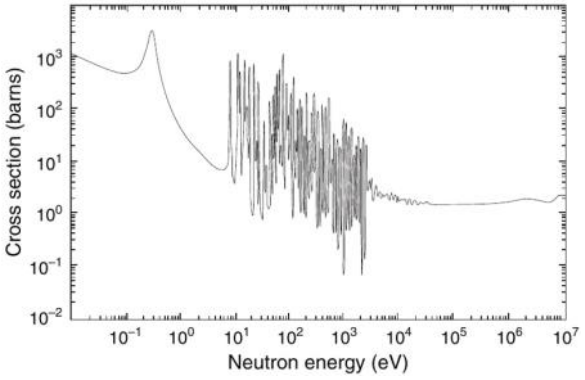
threshold values and lower micro cross sections make them less significant in basic reactor physics.

2.4.4. Fissionable Materials

- Incident neutrons of any energy cause fission in a fissile material. Energy dependence of cross-sections for the only naturally occurring fissile nuclide (^{235}U) illustrated in the figure.
- Fission in ^{238}U is a threshold reaction, whereas ^{238}U is a fertile material that transforms into ^{239}Pu, which exhibits a broad-spectrum fission cross-section.

Fission cross section of ^{235}U	Fission cross section of ^{239}Pu
	

- Thorium-232, like uranium-238, is a naturally occurring fertile isotope that, after neutron capture, transforms into fissile uranium-233 with a fission cross-section similar to previous figures.

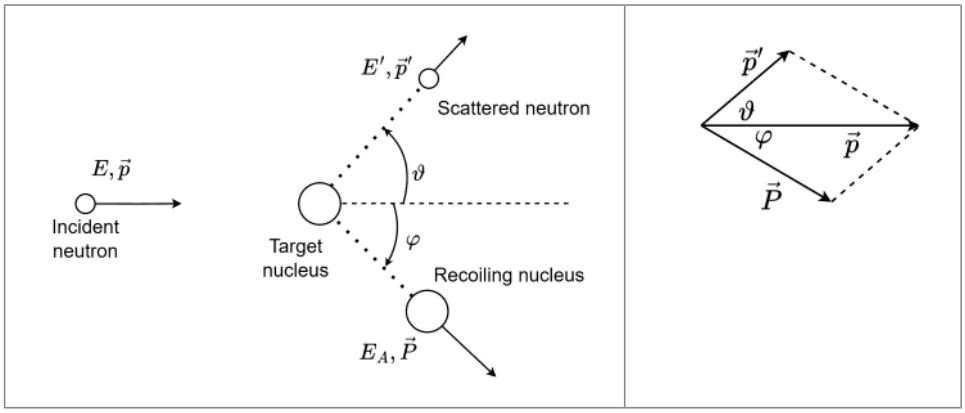
2.5. Neutron Scattering

2.5.0. Introduction

- Neutron energy spectrum in a reactor is determined by the competition between **scattering** and **absorption** reactions.
- Scattering collisions result in neutron energy degradation, referred to as "**neutron slowing down**" for neutrons significantly above thermal energy. Neutrons near thermal equilibrium can either gain or lose energy as they interact with thermal motions of surrounding nuclei.
- In a medium with a large ratio of scattering to absorption cross sections, the neutron spectrum is close to thermal equilibrium, known as a **"soft" or thermal spectrum**.
- In a system with a small ratio of neutron degradation to absorption, neutrons are absorbed before significant slowing down occurs, resulting in a **"hard" or fast spectrum**.

2.5.1. Elastic Scattering

Simplified method of Energy Loss determination



- Energy conservation: $E = E' + E_A$
- Momentum conservation: $\mathbf{p} = \mathbf{p}' + \mathbf{P}$
- Law of cosine: $P^2 = p^2 + (p')^2 - 2pp'\cos\vartheta$

$$\Rightarrow ME_A = mE + mE' - 2m\sqrt{EE'}\cos\vartheta \quad [\Leftarrow p^2 = 2mE]$$

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$$\Rightarrow AE_A = E + E' - 2\sqrt{EE'}\cos\vartheta \left[\Leftarrow \frac{M}{m} \approx A\right]$$

$$\Rightarrow (A+1)E' - 2\sqrt{EE'}\cos\vartheta - (A-1)E = 0 \ [\Leftarrow E = E' + E_A]$$

$$\Rightarrow \sqrt{E'} = \frac{2\sqrt{E}\cos\theta \pm \sqrt{4E\cos^2\theta + 4(A+1)(A-1)E}}{2(A+1)}$$

$$\Rightarrow \sqrt{E'} = \frac{\sqrt{E}}{A+1}\left[\cos\theta + \sqrt{\cos^2\theta + A^2 - 1}\right] \left[\because \sqrt{E'} \geq 0\right]$$

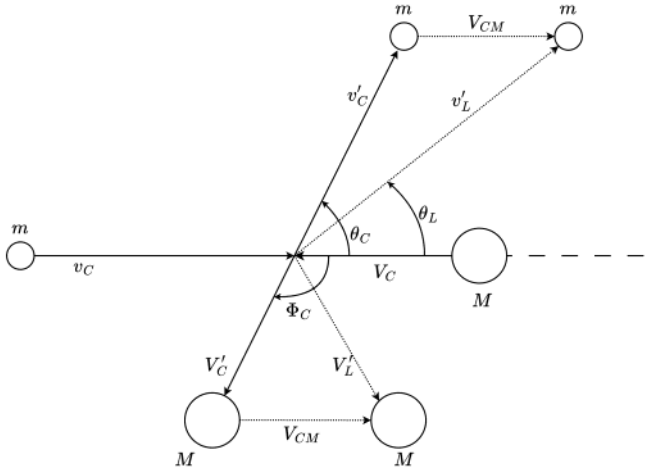
$$\therefore E' = \frac{E}{(A+1)^2}\left[\cos\theta + \sqrt{A^2 - \sin^2\vartheta}\right]^2$$

Kinematics of Elastic Scattering

- Basic principles
  - In elastic scattering, both the kinetic energy and momentum of the system are conserved.
  - In the center of mass frame, linear momentum of the system is zero before and after collision.
- Variable notations

	Before Collision		After collision	
Parameter	Lab (L)	CM (C)	Lab (L)	CM (C)
Neutron speed	v_L	v_C	v_L'	v_C'
Nuclide speed	V_L	V_C	V_L'	V_C'
Speed of CM	V_{CM}	0	V_{CM}	0
Neutron scattering angle	-	-	θ_L	θ_C
Nuclide scattering angle	-	-	Φ_L	Φ_C

- Collision diagram (L frame velocities are in dotted lines)



- In C frame
  - $mv_C + MV_C = 0 \Rightarrow V_C = -\frac{m}{M}v_C = -\frac{1}{A}v_C \left[\because \frac{M}{m} = A\right]$
  - $mv_C' + MV_C' = 0 \ \& \ KE_C = KE_C' \Rightarrow v_C = v_C' \ \& \ V_C = V_C'$
  - $\vec{F}_{ext} = 0 \Rightarrow V_{CM} = V_{CM}'$
- Velocity transformations in the L & C frames
  - $V_L = 0 = V_C + V_{CM} \Rightarrow V_{CM} = -V_C = \frac{1}{A}v_C$
  - $v_L = v_C + V_{CM} = \left(1 + \frac{1}{A}\right)v_C \Rightarrow v_C = \frac{A}{A+1}v_L \ \& \ V_{CM} = \frac{1}{A+1}v_L$

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- From collision diagram
  - $v_L' \sin\theta_L = v_C' \sin\theta_C \ \ldots(i)$
  - $v_L' \cos\theta_L = V_{CM} + v_C' \cos\theta_C \quad \ldots(ii)$
- Relation between scattering angles in C & L frames $[(i)/(ii)]$
  - $\tan\theta_L = \dfrac{\sin\theta_C}{\frac{V_{CM}}{v_C'} + \cos\theta_C} = \dfrac{\sin\theta_C}{\frac{1}{A} + \cos\theta_C}$
    - $A = 1 \Rightarrow \tan\theta_L = \dfrac{2\sin\frac{\theta_C}{2}\cos\frac{\theta_C}{2}}{2\cos^2\frac{\theta_C}{2}} = \tan\dfrac{\theta_C}{2} \Rightarrow \theta_L = \dfrac{\theta_C}{2}$
      - ◇ $\theta_C \in [0,\pi] \Rightarrow \theta_L \in \left[0,\frac{\pi}{2}\right] \Rightarrow$ no back scattering in L system for Hydrogen
- Relation between energy before and after collision $[(i)^2 + (ii)^2]$
  - $v_L'^{\,2} = v_C'^{\,2} sin^2\theta_C + \left(V_{CM} + v_C' cos\,\theta_C\right)^2 = v_C'^{\,2} + V_{CM}^2 + 2V_{CM}\cdot v_C' \cdot \cos\theta_C$

    $\Rightarrow v_L'^{\,2} = v_C^2 + \dfrac{1}{A^2}v_C^2 + 2\dfrac{v_C}{A}\cdot v_C\cos\theta_C = v_C^2\dfrac{A^2 + 2A\cos\theta_C + 1}{A^2}$

    $\Rightarrow v_L'^{\,2} = \left[\dfrac{A}{A+1}v_L\right]^2 \dfrac{A^2 + 2A\cos\theta_C + 1}{A^2} \Rightarrow \dfrac{v_L'^{\,2}}{v_L^2} = \dfrac{E'}{E}$

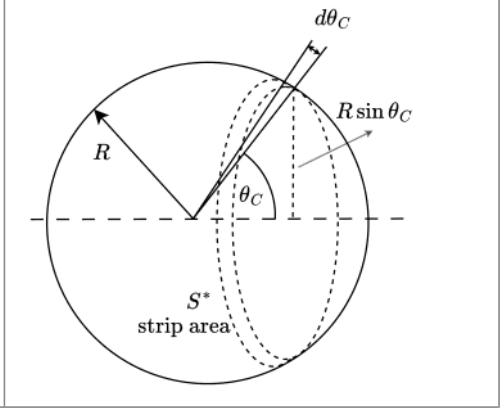
    $= \dfrac{A^2 + 2A\cos\theta_C + 1}{(A+1)^2} \quad \ldots(iiia)$

    $\therefore E' = E\cdot\dfrac{A^2 + 2A\cos\theta_C + 1}{(A+1)^2} \ \ldots(iiib)$
- Extreme cases
  1) $\theta_C = 0 \Rightarrow \cos\theta_C = 1 \Rightarrow E' = E$: No interaction → No energy loss
  2) $\theta_C = \pi \Rightarrow \cos\theta_C = -1 \Rightarrow E' = E\cdot\left(\dfrac{A-1}{A+1}\right)^2 \equiv \alpha E$
  - □ $\alpha = \left(\frac{A-1}{A+1}\right)^2$ – collision parameter: maximum relative energy loss in one collision
    - $1 + \alpha = \dfrac{2(A^2+1)}{(A+1)^2}; 1 - \alpha = \dfrac{4A}{(A+1)^2}$
  - □ Energy range for neutron after elastic scattering
    - $E' \in [\alpha E, E] \Leftrightarrow \alpha E \le E' \le E$
    - $A = 1$ (Hydrogen) $\Rightarrow \alpha = 0 \Rightarrow E' \in [0, E]$
- Energy change in terms of collision parameter
  - $E' = E\left[\dfrac{1+\alpha}{2} + \dfrac{1-\alpha}{2}\cos\theta_C\right] \quad \ldots(iv)$

Neutron Energy Distribution after Scattering

- As noted, neutron energy, E' lies in the interval $[\alpha E, E]$ after elastic scattering. To determine the probability $p(E \to E')dE'$ that energy following collision falls between E' and $E' + dE'$, let's consider the probability that scattering angle lies in between θ_C and $\theta_C + d\theta_C$ in the C frame.
  - $\omega(\theta_C)d\theta_C = \dfrac{S^*}{S} = \dfrac{2\pi R^2 \sin\theta_C\, d\theta_C}{4\pi R^2}$

    $\therefore \omega(\theta_C) = \frac{1}{2}\sin\theta_C$ – probability distribution function of scattering angles in C frame



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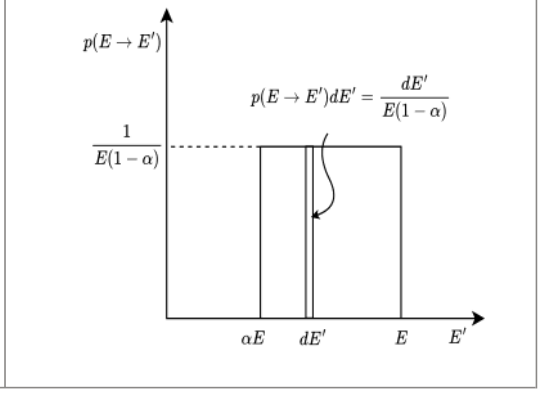
- Energy E' and scattering angle θ_C are uniquely related, thus
  - $p(E \to E')|dE'| = \omega(\theta_C)|d\theta_C| \Rightarrow p(E \to E') = \omega(\theta_C)\left|\frac{d\theta_C}{dE'}\right|$
    - $(iv) \Rightarrow \cos\theta_C = \frac{2E'}{E(1-\alpha)} - \frac{2}{1+\alpha} \Rightarrow -\sin\theta_C \frac{d\theta_C}{dE'} = \frac{2}{E(1-\alpha)}$
      $\Rightarrow \left|\frac{d\theta_C}{dE'}\right| = \frac{2}{E(1-\alpha)\cdot\sin\theta_C}$
  - $p(E \to E') = \omega(\theta_C)\left|\frac{d\theta_C}{dE'}\right| = \frac{1}{2}\sin\theta_C \cdot \frac{2}{E(1-\alpha)\cdot\sin\theta_C}$
    $\therefore p(E \to E') = \frac{1}{E(1-\alpha)}$

- Probability that scattered neutron's energy lies in the interval $[\alpha E, E]$
  - $P = \int_{\alpha E}^{E} p(E \to E')\,dE' = \frac{1}{E(1-\alpha)}\int_{\alpha E}^{E} dE' = 1$
- **Elastic scattering kernel**: Energy distribution for scattered neutrons
  - $p(E \to E')dE' = \begin{cases} \frac{dE'}{(1-\alpha)E} & if\ \alpha E \le E' \le E \\ 0 & \text{otherwise} \end{cases}$



- Mean Energy of Scattered Neutrons
  - Given the probability distribution function $p(E \to E')$ mean energy of scattered neutrons
    - $\overline{E'} = \frac{\int_{\alpha E}^{E} E'P(E \to E')dE'}{\int_{\alpha E}^{E} P(E \to E')dE'\ [=1]} = \frac{1}{E(1-\alpha)}\int_{\alpha E}^{E} E'dE'$
      $= \frac{1}{E(1-\alpha)}\left[\frac{E'^2}{2}\right]_{\alpha E}^{E}$
      $\Rightarrow \overline{E'} = \frac{1}{E(1-\alpha)}\left[\frac{E^2(1-\alpha^2)}{2}\right] = \frac{1+\alpha}{2}E$
  - This represents the arithmetic mean of the two endpoints $[\alpha E, E]$ since the probability distribution is constant across this range.
- Average Fractional Energy Loss in One Collision
  - Fractional energy loss
    - $\frac{\Delta E}{E} = \frac{E - E'}{E}$
  - Average fractional energy loss
    - $\overline{\left(\frac{\Delta E}{E}\right)} = \overline{\left(\frac{E-E'}{E}\right)} = \frac{\int_{\alpha E}^{E} \frac{E-E'}{E}\cdot P(E \to E')dE'}{\int_{\alpha E}^{E} P(E \to E')dE'\ [=1]}$
      $= \int_{\alpha E}^{E}\left(1 - \frac{E'}{E}\right)\cdot P(E \to E')dE'$
      $\Rightarrow \overline{\left(\frac{\Delta E}{E}\right)} = \int_{\alpha E}^{E} P(E \to E')dE' - \frac{1}{E}\int_{\alpha E}^{E} E'\cdot P(E \to E')dE' = 1 - \frac{1}{E}\overline{E'}$
      $= 1 - \frac{1+\alpha}{2}$

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$$\therefore \overline{\left(\frac{\Delta E}{E}\right)} = \frac{1-\alpha}{2} = \frac{2A}{(A+1)^2}$$

- Hydrogen: $A = 1 \Rightarrow \overline{\left(\frac{\Delta E}{E}\right)} = \frac{1}{2} = 50\% -$ neutrons, on average, lose half of its energy in **elastic scattering** with protons
- Graphite: $A = 12 \Rightarrow \overline{\left(\frac{\Delta E}{E}\right)} = 0.142 = 14.2\%$
- Uranium: $A = 238 \Rightarrow \overline{\left(\frac{\Delta E}{E}\right)} = 0.0083 \approx 1\%$

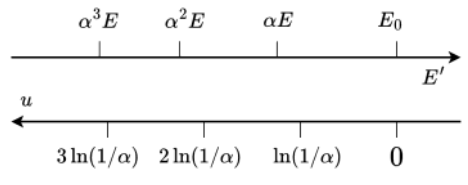
Average Logarithmic Energy Decrement

- **Lethargy**: Vast energy range for neutron slowing down and the fact that, average fractional energy loss is independent of energy leads to the introduction of a new variable lethargy defined as $\mathrm{u} = \ln\frac{\mathrm{E_0}}{\mathrm{E}}$ where E_0 is some reference value (typically $E_0 \coloneqq E_{\max} = 10\ MeV$).
  - $-\frac{\Delta E}{E} = -\Delta \ln E \neq f(E) \Rightarrow du \overset{\text{def}}{=} -\frac{dE}{E} = -d\ln E \Rightarrow u = -\int_{E_0}^{E} d\ln E = \ln\frac{E_0}{E}$
  - $E \downarrow \Rightarrow u \uparrow \Rightarrow$ fast neutrons become lethargic (sluggish) as neutrons lose energy in elastic scattering.
- Increase in lethargy as neutron loses energy ($E \to E'$)
  - $\Delta u = u' - u = \ln\frac{E_0}{E'} - \ln\frac{E_0}{E} = \ln\frac{E}{E'}$
- Average increase in lethargy per collision/ average logarithmic energy decrement/ slowing down decrement, ξ
  - $\xi \overset{\text{def}}{=} \overline{\Delta u} = \overline{\ln\frac{E}{E'}} = \frac{\int_{\alpha E}^{E} \ln\frac{E}{E'} \cdot P(E \to E')dE'}{\int_{\alpha E}^{E} P(E \to E')dE' \ [=1]} = \frac{1}{E(1-\alpha)}\int_{\alpha E}^{E} \ln\frac{E}{E'}dE'$

    $\Rightarrow \xi = \frac{1}{1-\alpha}\int_{E}^{\alpha E} \ln\left(\frac{E'}{E}\right) d\left(\frac{E'}{E}\right) = \frac{1}{1-\alpha}\left[\frac{E'}{E}\ln\frac{E'}{E} - \frac{E'}{E}\right]_{E}^{\alpha E}$

    $= \frac{1}{1-\alpha}[(\alpha\ln\alpha - \alpha) + 1]$

    $\therefore \xi = 1 + \frac{\alpha}{1-\alpha}\ln\alpha = 1 + \frac{(A-1)^2}{2A}\ln\frac{A-1}{A+1} \approx \begin{cases} \frac{2}{A+2/3} \text{ when } A > 3 \\ \frac{2}{A} \text{ when } A > 10 \end{cases}$
  - The slowing down decrement is independent of the energy of the scattered neutron and constant for a given nuclide!
  - For hydrogen, $A = 1 \Rightarrow \alpha = 0 \Rightarrow \xi = 1 + \frac{0}{1}\cdot(-\infty)$ indeterminate form
    - $x \coloneqq \frac{1}{\alpha} \Rightarrow \xi = 1 + \frac{1}{1-x}\ln x, \alpha \to 0 \Rightarrow x \to \infty$
    - $\xi = 1 + \lim_{x\to\infty}\frac{\ln x}{1-x} \overset{\text{L' Hospital}}{=} 1 + \lim_{x\to\infty}\frac{\frac{1}{x}}{-1} = 1 - 0 = 1$
- Maximum energy loss per collision in elastic scattering and corresponding maximum increase in lethargy.

$(\Delta E)_{\max} = E(1-\alpha) = f(E)$ $(\Delta u)_{\max} = \ln\frac{E_0}{\alpha E} - \ln\frac{E_0}{E} = \ln\frac{1}{\alpha} \neq f(E)$ Scattering process in energy variable is non-uniform but in lethargy variable it's uniform	

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(same interval between lethargies)	

- From lethargy scale it's easy to calculate average number of collision n required to slow neutron from energy E_1 to energy E_2
  - $n = \frac{u_2 - u_1}{\xi} = \frac{\ln\frac{E_0}{E_2} - \ln\frac{E_0}{E_1}}{\xi} = \frac{\ln\frac{E_1}{E_2}}{\xi}$
  - Required collisions to thermalize fast neutrons [$E_1 \coloneqq 2MeV, E_2 \coloneqq 0.025\ eV \Rightarrow \ln\frac{E_1}{E_2} = 18.2$]

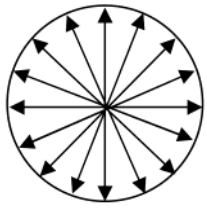
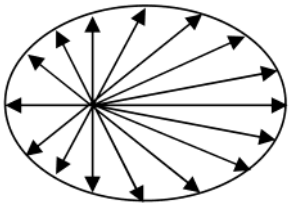
Nuclide	$\alpha = \left[\frac{A-1}{A+1}\right]^2$	$\xi = 1 + \frac{\alpha}{1-\alpha}\ln\alpha$	$n = \frac{\ln\frac{E_1}{E_2}}{\xi}$
^1H	0	1	18
$^2H/D$	0.111	0.725	25
^9Be	0.640	0.207	88
^{12}C	0.716	0.158	115
^{238}U	0.9833	0.008	2172

    - For thermal reactors lightweight materials are desirable and for fast reactors undesirable.
- Average slowing down decrement (SDD) for mixtures.
  - $\bar{\xi} = \frac{\sum_i \xi_i \Sigma_{si}}{\sum_i \Sigma_{si}}$
  - SDD for water
    - $\xi_{H_2O} = \frac{\xi_H \Sigma_s^H + \xi_O \Sigma_s^O}{\Sigma_s^{H_2O}} = \frac{\xi_H 2N_{H_2O}\sigma_s^H + \xi_O N_{H_2O}\sigma_s^O}{N_{H_2O}(2\sigma_s^H + \sigma_s^O)} = \frac{2\xi_H\sigma_s^H + \xi_O\sigma_s^O}{2\sigma_s^H + \sigma_s^O}$

      $\therefore \xi_{H_2O} = \frac{2\cdot 1\cdot 20b + \frac{2}{16}\cdot 4b}{2\cdot 20b + 4b} \approx 0.920$
    - $\xi_{D_2O} = \frac{2\xi_D\sigma_s^D + \xi_O\sigma_s^O}{2\sigma_s^D + \sigma_s^O} = \frac{2\cdot 0.725\cdot 4b + \frac{2}{16}\cdot 4b}{2\cdot 4b + 4b} \approx 0.525$

Anisotropy of Scattering in L System

- From the collision diagram it's apparent that $\theta_L < \theta_C$ due to the component V_{CM} and neutrons show preferrential forward scattering in L system, although in C system scattering is spherically symmetric or isotropic.

	
Isotropic	Anisotropic

- Spherical symmetry can be expressed as the average value of the cosines of all possible scattering angles in space. First let's establish a relation between $\cos\theta_L$ with $\cos\theta_C$
  - $(ii) \Rightarrow \frac{sin^2\,\theta_L}{sin^2\,\theta_C} = \frac{v_C'^{\,2}}{v_L'^{\,2}} = \frac{\left(\frac{A}{A+1}\right)^2 v_L^2}{v_L^2 \cdot \frac{A^2 + 2A\,cos\,\theta_C + 1}{(A+1)^2}} = \frac{A^2}{A^2 + 2A\,cos\,\theta_C + 1}$

    $\Rightarrow sin^2\,\theta_L = 1 - cos^2\,\theta_L = \frac{A^2\,sin^2\,\theta_C}{A^2 + 2A\,cos\,\theta_C + 1}$

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$$\Rightarrow cos^2\theta_L = 1 - \frac{A^2 - A^2\,cos^2\theta_C}{A^2 + 2A\,cos\,\theta_C + 1} = \frac{\left(1 + \mathrm{A}\cos\theta_C\right)^2}{A^2 + 2A\,cos\,\theta_C + 1}$$

$$\therefore \cos\theta_L = \frac{1 + A\cos\theta_C}{\sqrt{1 + A^2 + 2A\,cos\,\theta_C}} = f(\theta_C)$$

- To find the average of $\cos\theta_L = f(\theta_C)$ we use the PDF of θ_C: $\omega(\theta_C) = \frac{1}{2}\sin\theta_C$
  - $\overline{f(\theta_C)} = \overline{\cos\theta_L} = \int_0^\pi f(\theta_C)\cdot\omega(\theta_C)d\theta_C$

    $= \int_0^\pi \frac{1 + \mathrm{A}\cos\theta_C}{\sqrt{1 + A^2 + 2A\,cos\,\theta_C}}\left(\frac{1}{2}\sin\theta_C\right)d\theta_C$

    - $x = \cos\theta_C \Rightarrow \overline{\cos\theta_L} = \frac{1}{2}\int_{-1}^{1}\frac{1 + Ax}{\sqrt{1 + A^2 + 2Ax}}dx$
    - $u^2 = 1 + A^2 + 2Ax \Rightarrow dx = \frac{udu}{A}; \begin{bmatrix} x = -1 \Rightarrow u = A - 1 \\ x = 1 \Rightarrow u = A + 1 \end{bmatrix}; 1 + Ax$

      $= \frac{u^2}{2} - \frac{A^2 - 1}{2}$

    $\Rightarrow \overline{\cos\theta_L} = \int_{A-1}^{A+1}\frac{u^2du}{2A} - \frac{A^2 - 1}{2A}\int_{A-1}^{A+1}du = \frac{u^3}{6A}\Big|_{A-1}^{A+1} - \frac{A^2 - 1}{2A}\cdot u\Big|_{A-1}^{A+1}$

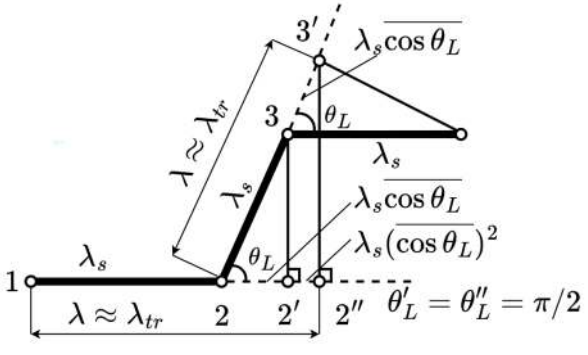
    $\therefore \overline{\cos\theta_L} = \frac{(A+1)^3 - (A-1)^3}{6A} - \frac{A^2 - 1}{A} = \frac{6A^2 + 2 - 6A^2 + 6}{6A} = \frac{2}{3\mathrm{A}}$

  - $A = 1 \Rightarrow \overline{cos\,\theta_L} = \frac{2}{3} \Rightarrow$ maximum anisotropic scattering with hydrogen
  - $\mathrm{A} \to \infty \Rightarrow \overline{\cos\theta_\mathrm{L}} \to 0 \Rightarrow$ anisotropy vanishes in heavy nuclides
- Note that the average of cosine in case of spherically symmetric scattering, as in C frame, is 0.
  - $\overline{\cos\theta_\mathrm{C}} = \int_0^\pi \cos\theta_C\left(\frac{1}{2}\sin\theta_C\right)d\theta_C = \frac{1}{4}\int_0^\pi \sin 2\theta_C = \frac{1}{8}\cos 2\theta_C\Big|_\pi^0 = \frac{1-1}{8}$

    $= 0$
  - If every scattering angle θ is assumed to be 90° then scattering becomes isotropic. ($\overline{\cos\theta} = 0$.)

Transport Mean Free Path and Scattering Cross Section

- To simplify the analysis in diffusion theory, neutrons are assumed to scatter isotropically, i.e. at 90° in the L frame.
- To transform anisotropic scattering into isotropic, transport mean free path λ_{tr} is introduced, which takes into account the increased effective mean free path due to forward scattering tendency of neutrons.

○ If neutron scatters from 2 to 3 with $\theta_L < \frac{\pi}{2}$, then we can assume it to scatter from 2′ with $\theta_L' = \frac{\pi}{2}$ so as to keep the scattering isotropic. ○ This increases the MFP λ_s to $\lambda_s + \lambda_s\,\overline{\cos\theta_L}$, and if we continue to coerce the scatterings in 90° angles, then the effective increase in MFP will be:	

$$\Rightarrow \lambda_{tr} = \lambda_s + \lambda_s\,\overline{\cos\theta_L} + \lambda_s\left(\overline{\cos\theta_L}\right)^2 + \cdots = \frac{\lambda_s}{1 - \overline{\cos\theta_L}} = \frac{\lambda_s}{1 - \frac{2}{3A}}$$

- Corresponding transport cross sections

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- $\Sigma_{tr} = \frac{1}{\lambda_{tr}} = \frac{1-\overline{\cos\theta_L}}{\lambda_s} = \Sigma_s\left(1-\overline{\cos\theta_L}\right) = \Sigma_s\left(1-\frac{2}{3A}\right) \Leftrightarrow \sigma_{tr} = \sigma_s\left(1-\overline{\cos\theta_L}\right)$.

2.5.3. Inelastic Scattering

- While elastic scattering cross sections are relevant for all neutron energies, inelastic scattering cross sections show threshold behavior, with heavies nuclides possessing lower threshold energies.
- To scatter inelastically, a neutron must elevate the target nucleus to one of its excited states and the energy of the scattered neutron depends on the excited energy level of the nuclide.
- Inelastic scattering differs from elastic scattering by the fact that excitation energy ($Q < 0$) must be introduced in the energy conservation equation. Momentum conservation remains the same both in L and C frames.
    - Momentum conservation in C frame
        - $mv_c + MV_C = 0 \Rightarrow V_C = -\frac{m}{M}v_C = -\frac{1}{A}v_C;\ \ mv_c' + MV_C' = 0 \Rightarrow V_C' = -\frac{1}{A}v_C'$
    - Velocity transformations in the L & C frames
        - $V_L = 0 = V_C + V_{CM} \Rightarrow V_{CM} = -V_C = \frac{1}{A}v_C$
        - $v_L = v_C + V_{CM} = \left(1+\frac{1}{A}\right)v_C \Rightarrow v_C = \frac{A}{A+1}v_L \,\&\, V_{CM} = \frac{1}{A+1}v_L$
    - Energy balance in C frame
        - $\frac{1}{2}mv_C^2 + \frac{1}{2}MV_C^2 = \frac{1}{2}m{v_C'}^2 + \frac{1}{2}M{V_C'}^2 - Q$
          $\Rightarrow v_C^2 + \frac{\mathrm{M}}{\mathrm{m}}V_C^2 + \frac{2\mathrm{Q}}{\mathrm{m}} = {v_C'}^2 + \frac{\mathrm{M}}{\mathrm{m}}{V_C'}^2 \Rightarrow v_C^2\left(1+\frac{1}{\mathrm{A}}\right) + \frac{2\mathrm{Q}}{\mathrm{m}} = {v_C'}^2\left(1+\frac{1}{\mathrm{A}}\right)$
          $\Rightarrow {v_C'}^2 = v_C^2 + \frac{2Q}{m}\frac{A}{A+1} = \left(\frac{A}{A+1}v_L\right)^2 + \frac{2Q}{m}\frac{A}{A+1}$
          $= \left(\frac{A}{A+1}\right)^2 v_L^2\left(1+\frac{Q}{\frac{1}{2}mv_L^2}\frac{A+1}{A}\right)$
    - From collision diagram we found that
        - ${v_L'}^2 = {v_C'}^2 + 2V_{CM}v_C'\cos\theta_C + V_{CM}^2$
          $= v_L^2\left\{\left(\frac{A}{A+1}\right)^2\left(1+\frac{Q}{\frac{1}{2}mv_L^2}\frac{A+1}{A}\right) + \frac{2A}{(A+1)^2}\sqrt{1+\frac{Q}{\frac{1}{2}mv_L^2}\frac{A+1}{A}}\cos\theta_C + \left(\frac{1}{A+1}\right)^2\right\}$
          $\Rightarrow E'$
          $= E\left\{\left(\frac{A}{A+1}\right)^2\left(1+\frac{Q}{E}\frac{A+1}{A}\right) + \frac{2A}{(A+1)^2}\sqrt{1+\frac{Q}{E}\frac{A+1}{A}}\cos\theta_C + \left(\frac{1}{A+1}\right)^2\right\}$
    - For E' to have real value, the quantity under the square root must be positive

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- $1+\frac{Q}{E}\frac{A+1}{A}\geq 0 \Rightarrow E \geq -\frac{A+1}{A}Q \Rightarrow E_{th}$
  $= -\frac{A+1}{A}Q$ [kinematic threshold]
- $E = E_{th} \Rightarrow E' = \frac{E}{(A+1)^2} \Rightarrow \Delta E = E - E' = E\left(1-\frac{1}{(A+1)^2}\right)$
- Fractional energy loss $\frac{\Delta E}{E} = 1 - \frac{1}{(A+1)^2}$
  - $^{12}C: A = 12 \Rightarrow \left(\frac{\Delta E}{E}\right)_{in} = 1 - \frac{1}{169} \approx 99.4\%$ whereas $\overline{\left(\frac{\Delta \mathrm{E}}{\mathrm{E}}\right)_{\mathrm{el}}}$
    $\approx 14.2\%$
  - $^{238}U: A = 238 \Rightarrow \left(\frac{\Delta E}{E}\right)_{in} = 1 - \frac{1}{239^2}$
    $\approx 99.998\%$ whereas $\overline{\left(\frac{\Delta \mathrm{E}}{\mathrm{E}}\right)_{\mathrm{el}}} \approx 1\%$

- Neutrons lose a significant amount of energy in inelastic collisions compared to elastic scattering. This is one of the reasons for using heavy metals in the core baffle to reduce the fast neutron fluence on the reactor vessel.

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3. Neutron energy distribution

Wednesday, September 27, 2023 8:03 AM

2.18.* prelude

$$p(E\to E') = \begin{cases} \frac{1}{(1-\alpha)E} & \alpha E < E' < E \\ 0 & \text{otherwise} \end{cases}$$ | Scattering probability distribution for a single nuclide

$\sigma_s(E\to E') \overset{def}{=} \sigma(E)\, p(E\to E')$ ⇝ prob. of scattering from E to E' per nuclide

$\overset{\times N}{\Rightarrow}\ \Sigma_s(E\to E') = \Sigma_s(E)\, p(E)$

for a mixture of nuclides

$$\Sigma_s^{mix}(E\to E') = \sum_i \Sigma_{si}(E\to E') = \sum_i N_i \sigma_{si}(E\to E')$$

$$= \sum_i N_i \sigma_{si}(E)\, p_i(E\to E') = \sum_i \Sigma_{si}(E)\, p_i(E\to E')$$

Again $\Sigma_s^{mix}(E\to E') = \Sigma_s^{mix}(E)\, p^{mix}(E\to E') = \sum_i \Sigma_{si}(E)\, p_i(E\to E')$

$$\Rightarrow p^{mix}(E\to E') = \frac{1}{\Sigma_s^{mix}} \sum_i \Sigma_{si}(E)\, p_i(E\to E')$$

with this probability distribution we can find averages of any quantity.

e.g. $\xi^{mix} = \overline{\ln \frac{E}{E'}} = \int \ln \frac{E}{E'}\, p^{mix}(E\to E')\, dE'$

$$= \frac{1}{\Sigma_s^{mix}(E)} \sum_i \Sigma_{si}(E) \int_{\alpha_i E}^{E} \ln\left(\frac{E}{E'}\right) p_i(E\to E')\, dE'$$

$$\boxed{\xi^{mix} = \frac{1}{\Sigma_s^{mix}(E)} \sum_i \Sigma_{si}(E)\, \xi_i}$$

to find $\left(\overline{\frac{E-E'}{E}}\right)^{mix}$

$$\left(\overline{\frac{E-E'}{E}}\right)^{mix} = \frac{1}{\Sigma_s^{mix}(E)} \sum_i \Sigma_{si}(E) \int_{\alpha_i E}^{E} \frac{E-E'}{E} \cdot \frac{1}{(1-\alpha_i)E}\, dE'$$

$$\boxed{\left(\overline{\frac{E-E'}{E}}\right)^{mix} = \frac{1}{\Sigma_s^{mix}(E)} \sum_i \Sigma_{si}(E)\, \frac{1}{2}(1-\alpha_i)}$$

$$\frac{1}{(1-\alpha_i)E}\left[1 - \frac{E'}{2E}\right]_{\alpha_i E}^{E}$$

$$= \frac{1}{(1-\alpha_i)E}\left[(1-\alpha_i)E - \frac{1}{2}(1-\alpha_i^2)E\right]$$

$$= 1 - \frac{1}{2}(1+\alpha_i) = \frac{1}{2}(1-\alpha_i)$$

e.g. $\left(\overline{\frac{E-E'}{E}}\right)_{H_2O} = \frac{1}{\Sigma_s^{H_2O}} \frac{1}{2}\left[\Sigma_s^H(1-\alpha_H) + \Sigma_s^O(1-\alpha_O)\right]$

$$= \frac{1}{N^{H_2O}(2\sigma_s^H + \sigma_s^O)} \frac{1}{2}\left[2N^{H_2O}\sigma_s^H(1-\alpha_H) + N^{H_2O}\sigma_s^O(1-\alpha_O)\right]$$

(σ_s^H: 20b; $1-\alpha_H$: 0; σ_s^O: 4b; $1-\alpha_O$: $(\frac{15}{17})^2$)

$$\left(\overline{\frac{E-E'}{E}}\right)_{H_2O} \simeq 0.465 = 46.5\% \quad \text{cf.} \quad \left(\overline{\frac{E-E'}{E}}\right)_H = 50\%$$

3. Neutron distributions in energy.

3.1. Introduction

$k = \frac{\#\text{ of neutrons in } i+1 \text{ gen}}{}$

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3.1. Introduction

$$k = \frac{\#\text{ of neutrons in } i+1 \text{ gen}}{\#\text{ of neutrons in } i\text{-th gen}}$$

- Number of neutrons $= f(\vec{r}, E, \vec{\Omega}, t)$
- Most detailed neutron distribution density, $n(\vec{r}, E, \vec{\Omega}, t)$ $[cm^{-3} eV^{-1} str^{-1} s^{-1}]$
- central focus here on E variable.

3.2. Nuclear fuel properties

Number of fission neutrons per neutron absorbed

$$\eta(E) = \frac{\nu \Sigma_f(E)}{\Sigma_a(E)} \quad ; \quad \Sigma_a(E) = \Sigma_f(E) + \Sigma_\gamma(E)$$

ν – # of neutrons produced per fission.

Single fissile nuclide (HEU)

$$\eta(E) = \frac{\nu N \sigma_f(E)}{N \sigma_a(E)} = \frac{\nu \sigma_f(E)}{\sigma_a(E)}$$

e.g HEU as in nuclear submarines/nuclear weapons

Civilian reactors – fissile (fi) + fertile (fe)

$$\tilde{e} = \frac{N_{fi}}{N_{fi} + N_{fe}} \quad ; \quad 1 - \tilde{e} = \frac{N_{fe}}{N_{fi} + N_{fe}} \quad ; \quad \tilde{e} \text{ - enrichment}$$

$$\Rightarrow \eta(E) = \frac{\nu \Sigma_f}{\Sigma_a} = \frac{\nu N_{fi} \sigma_f^{fi}(E) + \nu N_{fe} \sigma_f^{fe}(E)}{N_{fi} \cdot \sigma_a^{fi}(E) + N_{fe} \sigma_a^{fe}(E)}$$

$$\Rightarrow \eta(E) = \frac{\nu \tilde{e} \sigma_f^{fi}(E) + \nu (1-\tilde{e}) \sigma_f^{fe}(E)}{\tilde{e} \sigma_a^{fi}(E) + (1-\tilde{e}) \sigma_a^{fe}(E)}$$

Graphs in the book & Discussion.

problem 3.4.

3.3. Neutron moderators.

characteristics:
1. slowing down decrement (SDD), ξ
2. Slowing down power (SDP), $\xi \Sigma_s$
3. Moderating ratio (MR), $\frac{\xi \Sigma_s}{\Sigma_a}$

- $\xi \uparrow$ but $\Sigma_s \downarrow$ → not enough scattering to slow down (e.g. He: $\sigma_s \uparrow$, $N \downarrow \rightarrow \Sigma_s \downarrow$)
- $\xi \Sigma_s \uparrow$ but $\Sigma_a \uparrow$ → not good moderator (^{10}B)

calculation of moderator characteristics example

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#10

	σ_s, b	σ_a, b
$_1H$	20	0.3
$_2H$	4	0.0003
$_8O$	4	0.0001

3.4. Neutron Energy Spectra

- competition between scattering & absorption determines the energy distribution of neutrons.
- soft / thermal spectrum : avg energy loss per scattering ↑ ($\downarrow$ and (logical) $\wedge$ $\frac{\Sigma_s}{\Sigma_a}$ ↑)
- hard / fast spectrum : $\frac{\text{neutron degradation}}{\text{absorption}}$ ↓

Few Definitions

$\tilde{n}(E)$ $\left[\frac{\#}{cm^3 eV}\right]$ - density distribution function

$\Rightarrow \tilde{n}\,dE$ = # of neutrons/cm^3 with energies between E and E+dE (E → E+dE)

$\Rightarrow n = \int_0^\infty \tilde{n}(E)\,dE$ $\left[\frac{\#}{cm^3}\right]$

- Neutron flux distribution

$$\varphi(E) = v(E)\,\tilde{n}(E) \quad ; \quad v(E) = \sqrt{2mE} \quad \left[\frac{cm}{s}\right]$$

Physical interpretation: $\varphi(E)dE$ = total distance travelled in one second by all neutrons in 1 cm^3 with energies between E and E+dE.

- Interpretation of Macroscopic XS:

Σ_x = Probability of x-type reaction for a neutron with energy E per cm of flight.

- Reaction rate density / collision rate density

$$F_x(E) = \Sigma_x(E)\,\varphi(E) \qquad \frac{\text{Probable \# of x-reaction}}{s \cdot cm^3 \cdot eV}$$

$$\Rightarrow F_x = \int_0^\infty \Sigma_x(E)\,\varphi(E)\,dE \quad \left[\frac{\#}{cm^3 s}\right] \quad ; \; x = \begin{cases} a - \text{absorption} \\ s - \text{scattering} \\ f - \text{fission} \end{cases}$$

Infinite medium spectrum equation

consider balance equation in the Energy space

- Any collision removes neutrons from energy E
+ Neutron gain at Energy E < scattering from source with E.

Balance eqn : loss rate density = gain rate density

$$\Rightarrow \Sigma_t(E)\,\varphi(E) = \int p(E' \to E)\,\Sigma_s(E')\,\varphi(E')\,dE' + S(E)$$

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- For fission gain : $S(E) = \chi(E)\, S_f$; S_f = # of fission neutrons produced /cm³/s

- For elastic scattering

single nuclide case: $p(E' \to E) = \begin{cases} \frac{1}{(1-\alpha)E'} & \alpha E' \le E \le E' \\ 0 & \text{otherwise} \end{cases}$

composite case : $p(E' \to E) = \dfrac{\sum_i \Sigma_{si}(E') \cdot p_i(E' \to E)}{\sum_i \Sigma_{si}(E')}$

Definition: $\Sigma_s(E' \to E) = p(E' \to E)\, \Sigma_s(E')$

Analysis of fission spectrum with the balance equation.

Fast neutrons : $0.1\ MeV \le E$

No fission neutrons $\chi(E)=0$ { Intermediate neutrons : $1eV \le E < 0.1\ MeV$ { No upscattering $\Sigma_s(E' \to E) = 0$, $E' < E$

Thermal range : $E < 1\ eV$

Fast neutrons

$$\Sigma_t(E)\,\varphi(E) = \int \Sigma_s(E' \to E)\,\varphi(E')\,dE' + \chi(E)\, S_f \qquad E > 0.1\ MeV$$

(integral: even single scattering removes from the E range; $\chi(E) S_f$: dominant term)

$$\Rightarrow \quad \varphi(E) \approx \frac{\chi(E)\, S_f}{\Sigma_t(E)}$$

- rough approximation of the flux.
- only uncollided fission neutrons.

↓ fast spectrum

- low Z materials degrades the spectrum.

Intermediate / Slowing down range

Slowing down in an infinite hydrogen medium

- Simplifying assumption: Dirac's Delta function

① $S(E) = S^* \delta(E - E_0)$ - all neutrons are introduced at energy E_0.

② $\Sigma_s(E' \to E) = 0$ for $E' < E$: No upscattering.

③ $\Sigma_a = 0 \Rightarrow \Sigma_t = \Sigma_s$: No absorption.

- From Balance equation.

$$\Sigma_s(E)\,\varphi(E) = \int_E^\infty \Sigma_s(E' \to E)\,\varphi(E')\,dE' + S^* \delta(E - E_0)$$

For isotropic scattering $\Sigma_s(E' \to E) = \dfrac{\Sigma_s(E')}{(1-\alpha)E'} \underset{\alpha=0}{=} \dfrac{\Sigma_s'(E)}{E'}$

$E < E_0 \Rightarrow$ $\Sigma_s(E)\,\varphi(E) = \int_E^{E_0} \dfrac{\Sigma_s(E')\,\varphi(E')}{E'}\,dE'$ | $F_s^H(E) = \Sigma_s^H(E)\,\varphi(E)$ collision rate density

$$\Rightarrow \quad F_s(E) = \int_E^{E_0} \frac{F_s(E')}{E'}\,dE' \qquad \text{Integral equation}$$

$$\Rightarrow \quad \frac{d}{dE} F_s(E) = -\frac{F_s(E)}{E} \qquad \left[\text{Liebniz integral rule}: \frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)\,dt\right) = f(x,b(x)) \cdot \frac{d}{dx} b(x) - f(x,a(x)) \cdot \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt\right]$$

$$\Rightarrow \left[\frac{d}{dE} F_s(E)\right] E + F_s(E) = \frac{d}{dE}\left(F_s(E) \cdot E\right) = 0$$

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$\Rightarrow \left|\frac{d}{dE} F_s(E)\right| E + F_s(E) = \frac{d}{dE}\left(F_s(E)\cdot E\right) = 0$ $\ominus f(x,a(x))\cdot\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)\,dt$.]

$\Rightarrow \quad F_s(E) = \Sigma_s(E)\,\varphi(E) = \frac{c}{E}$ (c: constant) $\sim F_s^H(E) \sim \frac{1}{E}$ "one over E"

$\therefore \boxed{\varphi(E) = \frac{c}{\Sigma_s(E)\,E}}$ if $\Sigma_s(E) = \Sigma_s = \text{const} \quad \varphi \sim \frac{1}{E}$

flux in terms of lethargy, u

by def. $du = -\frac{dE}{E} \Rightarrow \left|\frac{dE}{du}\right| = E$

but, $|F(u)\,du| = |F(E)\,dE|$ – reaction density remains same

$\Rightarrow F(u) = F(E)\left|\frac{dE}{du}\right| = F(E)\,E$ [E = const]

Therefore,

$F_s(u) = \phi(u)\cdot\Sigma_s(u) = \text{const} \overset{\text{def}}{=} Q_0 \quad \Rightarrow \quad \phi(u) = \frac{Q_0}{\Sigma_s^H(u)}$

To determine Q_0, we introduce a new quantity called "slowing down density" $q(E)$

E'' E E'

$q(E)$ = # of neutrons slowing down past energy E per cm³ per s.

$$q(E) = \int_0^E dE'' \int_E^\infty \Sigma_s(E'\to E'')\,\phi(E')\,dE'$$

For the present problem,

$$q(E) = \int_0^E dE'' \int_E^{E_0} \Sigma_s(E'\to E'')\,\phi(E')\,dE' + \int_0^E dE'' \int_E^{E_0} \Sigma_s(E'\to E'')\,S(E')\,dE$$

For isotropic elastic scattering in hydrogen: $\Sigma_s^H(E'\to E'') = \frac{\Sigma_s^H(E')}{E'} \neq f(E'')$

$$\Rightarrow q(E) = E\int_E^{E_0} \frac{F_s(E')}{E'}\,dE' + E\int_E^{E_0} \frac{\Sigma_s^H(E')\,S^*\cdot\delta(E-E_0)}{E'}\,dE'$$

$$\Rightarrow q(E) = E\int_E^{E_0} \frac{Q_0}{(E')^2}\,dE' + \frac{E}{E_0}\Sigma_s^H(E_0)\,S_0$$

$$\therefore q(E) = E\left[\frac{1}{E} - \frac{1}{E_0}\right]Q_0 + \frac{E}{E_0}\Sigma_s^H(E_0)\,S_0$$

At source energy, E_0: $q(E_0) = \Sigma_s^H(E_0)\,S_0$ (all source neutrons ⌐ slows down past E.)

[Infinite medium with $\Sigma_a = 0 \Rightarrow q(E) = q(E_0)$

$$\Rightarrow \left[1 - \frac{E}{E_0}\right]Q_0 + \frac{E}{E_0}\Sigma_s^H(E_0)\,S_0 = \Sigma_s^H(E_0)\,S_0$$

$$\Rightarrow \left[1 - \frac{E}{E_0}\right]Q_0 = \left[1 - \frac{E}{E_0}\right]\Sigma_s^H(E_0)\,S_0 \Rightarrow \boxed{Q_0 = \Sigma_s^H(E_0)\,S_0}$$

Thus, for slowing down in an infinite nonabsorbing hydrogen medium

$$\varphi(E) = \frac{S_0\,\Sigma_s^H(E_0)}{E\,\Sigma_s^H(E)} \quad \Leftrightarrow \quad \varphi(u) = \frac{S_0\,\Sigma_s^H(u_0)}{\Sigma_s^H(u)}$$

Slowing down medium with $A > 1$ ($\alpha \neq 0$)

- In the energy range where upscatter is absent, all fission produced neutrons

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slows down past any energy E (in that range) unless absorbed before E. Thus

slowing down density, $q(E)$ [$cm^{-3}s^{-1}$]

$$q(E) = -\int_E^\infty \Sigma_a(E')\,\varphi(E')\,dE' + \int_E^\infty \chi(E')\,S_f\,dE' \quad ; \quad 1eV < E$$

- In the intermediate range ($1eV < E < 0.1\,MeV$) $\chi(E) \approx 0 \Rightarrow \int_E^\infty \chi(E')dE' \approx \int_0^\infty \chi(E')dE' = 1$

$$\Rightarrow q(E) = -\int_E^\infty \Sigma_a(E')\,\varphi(E')\,dE' + S_f \quad \Rightarrow \frac{d}{dE}q(E) = \Sigma_a(E)\,\varphi(E)$$

Thus, $\Sigma_a(E)\uparrow \Rightarrow q(E)\downarrow$ as $dE<0$; $\Sigma_a(E)=0 \Rightarrow q(E) = const.$

- Except for resonance energies, $\Sigma_a \simeq 0$ in the intermediate range $\Rightarrow q(E) = q = const.$

- Balance eqn in the intermediate range, ($\chi(E)=0$; $\Sigma_a = 0 \Rightarrow \Sigma_t = \Sigma_s$)

$$\underbrace{\Sigma_s(E)\varphi(E)}_{F_s(E)} = \int \underbrace{p(E'\to E)}_{\text{kernel}}\,\underbrace{\Sigma_s(E')\,\varphi(E')}_{F_s(E')}dE' \quad : \text{integral equation}$$

- If $E < E_{th}^{in}$ (in-inelastic scattering) and single moderator is present, we can use the elastic scattering kernel. Thus,

$$F_s(E) = \int_E^{E/\alpha} \frac{1}{(1-\alpha)E'}\,F_s(E')\,dE' \Leftrightarrow \frac{dF_s(E)}{dE} = \frac{1}{1-\alpha}\left[\frac{F_s(\frac{E}{\alpha})}{E/\alpha}\cdot\frac{1}{\alpha} - \frac{F_s(E)}{E}\right]$$

- due to the $F_s(E/\alpha)$ term, we can't solve the equation directly. [let's try our luck with the solⁿ form for $\Delta=1$]

$$F_s(E) := \frac{c}{E} \leftarrow \text{constant} \Rightarrow -\frac{c}{E^2} = \frac{1}{1-\alpha}\left[\underbrace{\frac{c}{E/\alpha}}_{=\alpha\frac{c}{E^2}}\cdot\frac{1}{\alpha} - \frac{c}{E^2}\right] = \frac{1}{1-\alpha}(\alpha-1)\frac{c}{E^2} = -\frac{c}{E^2} : \text{Identity !}$$

- To determine c we use the def of $q(E)$. Figure:

$$q(E) = \int_E^{E/\alpha}\left[\int_{\alpha E'}^{E}\underbrace{\frac{1}{(1-\alpha)E'}}_{p(E'\to E'')}\cdot\underbrace{\Sigma_s(E')\varphi(E')}_{F_s(E')}dE''\right]dE'$$

① # of neutrons that scatter at $E'(>E)$ to $E''(<E)$ fall in the interval $\alpha E' \le E'' \le E$ ← upper lim fix, lower lim vary.
② Only the neutrons in $[E, E/\alpha]$ can scatter below E.

$F_s(E') = \frac{c}{E'} \Rightarrow q(E) = \frac{c}{1-\alpha}\int_E^{E/\alpha}\left[\int_{\alpha E'}^{E}\frac{dE''}{E''^2}\right]dE'$

$$= \frac{c}{1-\alpha}\underbrace{\int_E^{E/\alpha}\frac{E-\alpha E'}{E'^2}dE'}_{I}$$

$$I = \int_E^{E/\alpha}\left(\frac{E}{E'^2} - \frac{\alpha}{E'}\right)dE' = \left[\frac{E}{E'} + \alpha\ln E'\right]_E^{E/\alpha} = \left[1-\alpha+\alpha\ln\frac{E}{E/\alpha}\right] = (1-\alpha)\underbrace{\left[1+\frac{\alpha}{1-\alpha}\ln\alpha\right]}_{\xi}$$

$\Rightarrow q(E) = q = c\xi \Rightarrow c = \frac{q}{\xi}$

Thus, $F_s(E) = \Sigma_s(E)\varphi(E) = \frac{c}{E} = \frac{q}{\xi E} \Rightarrow \boxed{\varphi(E) = \frac{q}{\xi\Sigma_s(E)E}}$

- For more multiple scattering nuclide : $\varphi(E) = \frac{q}{\bar{\xi}\,\Sigma_s(E)E}$; $\bar{\xi} = \frac{\sum_i \Sigma_s^i \xi_i}{\sum_i \Sigma_s^i}$

} "one-over E" flux in the intermediate/slowing down range.

※ In the lethargy variable [Prob. 3.7]

By def $u = \ln\frac{E_0}{E} \Rightarrow E = E_0 e^{-u}$ & $du = -\frac{dE}{E}$ ($dE<0$)

- flux in lethargy: $\varphi(u)\,du = -\varphi(E)\,dE \Rightarrow \varphi(u) = -\varphi(E)\frac{dE}{du} = E\varphi(E)$

- Macro cross sections in lethargy : $\Sigma_x(u) = \Sigma_x(E)$

- Kernel for elastic scattering in lethargy : $p(u\to u')du' = -p(E\to E')dE'$

$$\Rightarrow p(u\to u') = -p(E\to E')\frac{dE'}{du'} = p(E\to E')E' = \frac{E'}{(1-\alpha)E} = \frac{E_0 e^{-u'}}{(1-\alpha)E_0 e^{-u}}$$

$$\Rightarrow p(u\to u') = \frac{e^{u-u'}}{1-\alpha}$$

Now, $E' \le E \Rightarrow u' = \ln\frac{E_0}{E} = u$ and $E' = \alpha E \Rightarrow u' = \ln\frac{E_0}{\alpha E} = \ln\frac{E_0}{E} + \ln\frac{1}{\alpha} = u + \ln\frac{1}{\alpha}$.

$$p(u\to u') = \begin{cases}\frac{e^{u-u'}}{1-\alpha}, & u \le u' \le u + \ln\frac{1}{\alpha} \\ 0, & \text{otherwise.}\end{cases}$$

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- Collision density in lethargy, $F(u)$

$$F(u)\,du = -F(E)\,dE \Rightarrow F(u) = -\Sigma(E)\varphi(E)\frac{dE}{du} = \Sigma(E)\varphi(E)\cdot E$$

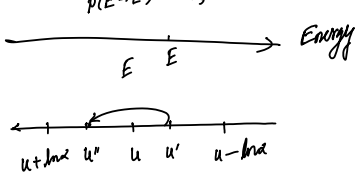
$$\Rightarrow F(u) = \Sigma(u)\varphi(u)$$

- Slowing down density, $q(u)$

$$q(u) = \int_{u-\Delta u}^{u} du' \int p(u' \to u'') \cdot F(u')\,du''$$

$$= \int_{u}^{u-\Delta u} \left[\int_{u'+\Delta u}^{u} \frac{\Sigma_s(u')\varphi(u')}{(1-\alpha)}\, e^{u'-u''}\, du'' \right] du'$$

$$q(E) = \int_{E}^{E/\alpha} \left[\int_{\alpha E'}^{E} \underbrace{\frac{1}{(1-\alpha)E'}}_{p(E'\to E'')} \cdot \underbrace{\Sigma_s(E')\varphi(E')}_{F_s(E')}\, dE'' \right] dE'$$



Energy Self-Shielding

- In presence of Resonance absorbers $\varphi(E) \neq \frac{c}{E}$

Rough estimate of the flux with resonance absorbers

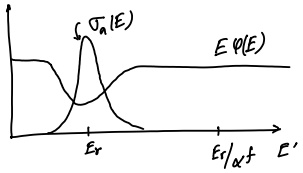
Medium: fuel (f) + Moderator (m)
Scattering: only elastic
Source: $\chi(E) = 0$
} approximations in the intermediate range

$$\Sigma_t(E)\varphi(E) = \int_{E}^{E/\alpha^f} \frac{1}{(1-\alpha^f)E'} \Sigma_s^f(E')\varphi(E')\,dE' + \int_{E}^{E/\alpha^m} \frac{1}{(1-\alpha^m)E'} \Sigma_s^m(E')\varphi(E')\,dE'$$

In Narrow resonance approximation (valid for most res. absorbers)

for most areas under the above integrals $\varphi(E) \sim \frac{1}{E}$

Thus ignoring absorption between resonance we can use



$\varphi(E) = \frac{q}{\xi \Sigma_s(E) E}$ in the integrals. Assuming Σ_s = const in the interval we determine by evaluating the integrals.

$$\varphi(E) = \frac{q}{\bar{\xi}\,\bar{\Sigma}_t(E)\,E} \qquad \Sigma_t = \Sigma_a + \Sigma_s$$

At energies, where absorption is high, flux decreases (depression in the figure)
This is referred to as energy self-shielding.

Thermal Neutrons

- In the thermal energy range, thermal neutron source : scattering source (neutron scattering down from higher energies)

Balance equation :

$$\Sigma_t(E)\varphi(E) = \int_{0}^{E_0} \Sigma_s(E' \to E)\varphi(E')dE' + \underbrace{\int_{E_0}^{\infty} \Sigma_s(E' \to E)\varphi(E')dE'}_{S(E)\,q_0 \text{ – thermal neutron source}} \qquad E < E_0$$

$S(E)$ for pure elastic scattering without absorption and $1/E$ flux assumption.

$$S(E)q_0 = \int_{E_0}^{\infty} \Sigma_s(E')\,p(E' \to E)\,\varphi(E')\,dE'$$

$$p(E' \to E) = \frac{1}{1-\alpha}\frac{1}{E'} \quad ; \quad \varphi(E) = \frac{q_0}{E\Sigma_s(E)E}$$

$$\Rightarrow \quad S(E) = \frac{1}{\xi(1-\alpha)} \int_{E_0}^{E/\alpha} \frac{dE'}{E'^2} = \frac{1}{\xi(1-\alpha)} \left[\frac{1}{E_0} - \frac{\alpha}{E} \right]$$

$$\Rightarrow \quad S(E) = \begin{cases} \frac{1}{\xi(1-\alpha)} \left[\frac{1}{E_0} - \frac{\alpha}{E} \right] & \alpha E_0 < E < E_0 \\ 0 & E < \alpha E_0 \end{cases}$$

Normalization of $S(E)$

$$\int_{\alpha E_0}^{E_0} S(E)\,dE = \frac{1}{\xi(1-\alpha)} \int_{\alpha E_0}^{E_0} \left(\frac{1}{E_0} - \frac{\alpha}{E} \right) dE = \frac{1}{\xi(1-\alpha)} \left[\frac{E}{E_0} - \alpha \ln E \right]_{\alpha E_0}^{E_0}$$

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* Normalization of $S(E)$

$$\int_0^{E_0} S(E)\,dE = \frac{1}{\xi(1-\alpha)} \int_{\alpha E_0}^{E_0} \left(\frac{1}{E_0} - \frac{\alpha}{E}\right) dE = \frac{1}{\xi(1-\alpha)} \left[\frac{E}{E_0} - \alpha \ln E\right]_{\alpha E_0}^{E_0}$$

$$= \frac{1}{\xi(1-\alpha)} \left[1 - \alpha - \alpha \ln \frac{1}{\alpha}\right] = \frac{1}{\xi(1-\alpha)} \underbrace{\left[1 + \frac{\alpha}{1-\alpha} \ln \alpha\right]}_{\xi} (1-\alpha)$$

$$\Rightarrow \int_0^{E_0} S(E)\,dE = 1$$

Again the balance equation.

$$\Sigma_t(E)\varphi(E) = \int_0^{E_0} \Sigma_s(E' \to E)\,\varphi(E')\,dE' + S(E) q_0$$

- here in the thermal range $\Sigma_s(E' \to E) = \Sigma_s(E')\, p(E' \to E)$ is difficult to represent. No simple scattering distribution. [Thermal motion + binding energy of nuclei to molecules need to be considered] (hard sphere collision not valid in this energy range)
- In an infinite medium, in absence of absorption, neutron population grows continuously. If after some time, $q_0 := 0$, then

$$\Sigma_s(E)\varphi(E) = \int_0^{E_0} \Sigma_s(E' \to E)\,\varphi(E)\,dE'$$

But from the principle of detailed balance (statistical theory)

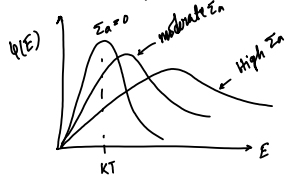
equilibrium: $\Sigma_s(E' \to E)\,\varphi(E) = \Sigma_s(E \to E')\,\varphi(E)$: upscatter = downscatter

and neutrons assume MB distribution: $\tilde{n}''(E) = n''' \frac{2\pi}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT}$

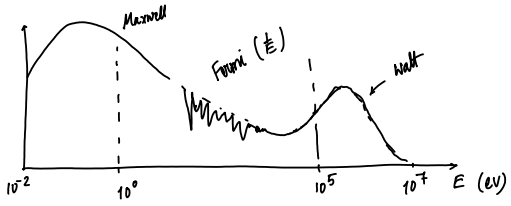
$$\varphi_M(E) = \tilde{n}'''(E) \cdot v = \tilde{n}''' \sqrt{\frac{2E}{m}} = AE e^{-E/kT} \quad ; \quad A = \frac{2\pi}{(kT)^{3/2}} \sqrt{\frac{2}{m}}$$

prove: $\int_0^{\infty} \varphi_M(E)\,dE = 1$

- If $\Sigma_a \neq 0$, full equilibrium is not reached: spectrum shift upward: spectral hardening.



Fast and Thermal Reactor spectra



3.5 Energy-Averaged Reaction Rates

- Average of a quantity x with distribution $f(x)$ in the domain X

$$\bar{x} = \frac{\int_X x f(x)\,dx}{\int_X f(x)\,dx}$$

Average Macroscopic section for x type reaction:

x = r.f.s $\quad \bar{\Sigma}_x = \dfrac{\int_0^{\infty} \Sigma_x(E)\varphi(E)\,dE}{\underbrace{\int_0^{\infty} \varphi(E)\,dE}_{\phi}}$ $\quad \begin{cases} [\varphi(E)] = \frac{\#}{cm^2 s\, eV} \\ [\phi] = \frac{\#}{cm^2 s} \end{cases}$

$\int_0^{\infty}$ — $= 1$

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$\int_0^\infty \varphi(E)\,dE = \phi$ $\quad [\varphi] = \frac{\#}{cm^2 s}$

$\Rightarrow \underbrace{\int_0^\infty \Sigma_x(E)\,\varphi(E)\,dE}_{\text{Reaction rate density}} = \bar{\Sigma}_x \phi$ $\quad [cm^{-1}]\,[\frac{\#}{cm^2 s}] \equiv [\frac{\#}{cm^3 s}]$

Microscopic Average Cross-section $\quad (\because \Sigma_x = \sigma_x N)$

$$\bar{\sigma}_x = \frac{\int_0^\infty \sigma_x(E)\,\varphi(E)\,dE}{\int_0^\infty \varphi(E)\,dE}$$

Dividing into energy ranges:

$$\int \sigma_x(E)\,\varphi(E)\,dE = \int_T \sigma_x(E)\,\varphi(E)\,dE + \int_I \sigma_x(E)\,\varphi(E)\,dE + \int_F \sigma_x(E)\,\varphi(E)\,dE$$

T: thermal, T: 0.001 eV – 1 eV; I - Intermediate, 1 eV – 0.1 MeV; F - Fast, 0.1 MeV – 10 MeV

$$\Rightarrow \bar{\sigma}_x \phi = \bar{\sigma}_{xT}\phi_T + \bar{\sigma}_{xI}\cdot\phi_I + \bar{\sigma}_{xF}\phi_F$$

Where
- flux in Ω range: $\phi_\Omega = \int_\Omega \varphi(E)\,dE$ $\quad \Omega = T, I, F$
- Average micro xs in Ω range: $\bar{\sigma}_{x\Omega} = \frac{\int_\Omega \sigma_x(E)\,\varphi(E)\,dE}{\phi}$

Fast cross-section averages

Fast flux, $\varphi(E) = \frac{\chi(E)\cdot S'''}{\Sigma_t(E)} \approx \chi(E)$ $\quad \left\{ \frac{S'''}{\Sigma_t(E)} (:=1) \right.$ Normalizing to 1

$$\bar{\sigma}_{xF} = \frac{\int_F \sigma_x(E)\,\varphi(E)\,dE}{\underbrace{\int_F \varphi(E)\,dE}_{=\chi(E),\ \approx 1}} \approx \int \sigma_x(E)\,\chi(E)\,dE$$

Resonance cross-section averages

Intermediate range flux, $\varphi(E) = \frac{q}{\xi \Sigma_t(E) E} \approx \frac{1}{E}$; $\left\{ \begin{array}{l} \text{Normalization} \\ \frac{q}{\xi \Sigma_t} := 1 \\ \text{energy independent} \end{array} \right.$

$$\bar{\sigma}_{xI} = \frac{\int_I \sigma_x(E)\,\varphi(E)\,dE}{\int_I \varphi(E)\,dE} = \frac{\int_I \sigma_x(E)\,\frac{dE}{E}}{\int_I \frac{dE}{E}} = 0.0869\, I_x$$

Where, $\left\{ \begin{array}{l} I_x = \int_I \sigma_x(E)\,\frac{dE}{E} \text{ - resonance integrals} \\ \left[\int_{1eV}^{10^5 eV} \frac{dE}{E}\right]^{-1} \approx 0.869 \end{array} \right.$

Thermal cross Section Averages

$$\bar{\sigma}_{xT} = \frac{\int_T \sigma_x(E)\,\varphi(E)\,dE}{\int_T \varphi(E)\,dE}$$

To do further analysis with thermal spectrum,

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the concept of Gamma function comes in handy.

Mathematical excursion: Gamma function

definition: $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$

properties.

① $\Gamma(1) = \int_0^\infty t^{1-1} e^{-t} dt = -e^{-t}\Big|_0^\infty = 1$

$\boxed{\Gamma(1) = 1}$

② $\Gamma(x+1) = \int_0^\infty t^{x+1-1} e^{-t} dt = \int_0^\infty \underbrace{t^x}_{u} \underbrace{d(-e^{-t})}_{dv}$

$= -t^x e^{-t}\Big|_0^\infty + x\int_0^\infty t^{x-1} e^{-t} dt$

0 (e^{-t} decays faster, t^x grows.) $\qquad \Gamma(x)$

$\boxed{\Gamma(x+1) = x\,\Gamma(x)}$

③ $\Gamma(4) = \Gamma(3+1) = 3\Gamma(3) = 3\Gamma(2+1) = 3.2\,\Gamma(2)$

$= 3.2.1.\underbrace{\Gamma(1)}_{=1} = 3!$

$\boxed{\Gamma(n) = (n-1)!}$

④ $\Gamma(\tfrac{1}{2}) = \int_0^\infty t^{-1/2} e^{-t} dt$

subs: $t = u^2 \quad t^{1/2} = \frac{1}{u}$, $dt = 2u\,du$

$= \int_0^\infty \frac{1}{u} e^{-u^2} \cdot 2u\,du$

$= 2\underbrace{\int_0^\infty e^{-u^2} du}_{\frac{\sqrt{\pi}}{2}}$

$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

$\boxed{\Gamma(\tfrac{1}{2}) = \sqrt{\pi}}$

Thermal spectrum

$\phi(E) = n(E) \cdot v(E)$

$= A\sqrt{\frac{2}{m}}\, E\, e^{-E/kT}$

$\begin{cases} n(E) = A \cdot \sqrt{E}\, e^{-E/kT} \\ v(E) = \sqrt{\frac{2E}{m}} \end{cases}$

Normalization: $\int_0^\infty \phi(E)\,dE = 1$

$\Rightarrow A\sqrt{\frac{2}{m}} \int_0^\infty E\, e^{-E/kT} dE = 1$

sub $E/kT = t \Rightarrow E = kT\,t \rightarrow dE = kT\,dt$

$\Rightarrow \sqrt{\frac{2}{m}} A (kT)^2 \underbrace{\int_0^\infty t\, e^{-t} dt}_{\Gamma(2) = 1\,\Gamma(1) = 1} = 1$

$\Rightarrow A = \sqrt{\frac{m}{2}} \frac{1}{(kT)^2} \Rightarrow \boxed{\phi(E) = \frac{1}{(kT)^2} E\, e^{-E/kT}}$

Thermal averaged absorption cross section

$\sigma_a(E) = \sigma_a(E_0)\sqrt{\frac{E_0}{E}}$ (1/v absorbers)

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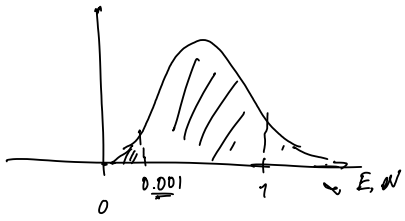
$$\bar{\sigma}_{aT} = \frac{\int_T \sigma_a(E)\,\varphi(E)\,dE}{\int_T \varphi(E)\,dE}$$

due to negligible contribution of the distribution beyond thermal range, we can extend the limits of integration to $0 \ldots \infty$



$$\bar{\sigma}_{aT} = \frac{\int_0^\infty \sigma_a(E_0)\sqrt{\frac{E_0}{E}}\,\frac{1}{(KT)^2}\,E\,e^{-E/kT}\,dE}{\underbrace{\int_0^\infty \frac{1}{(KT)^2}\,E\,e^{-E/kT}\,dE}_{=1}}$$

$$= \frac{\sigma_a(E_0)}{(KT)^2}\sqrt{E_0}\int_0^\infty \sqrt{E}\,e^{-E/kT}\,dE$$

[sub: $\frac{E}{KT} = t \quad \sqrt{E} = \sqrt{KT}\,t^{1/2}$

$\Rightarrow dE = KT\,dt$]

$$= \frac{\sigma_a(E_0)}{(KT)^{1/2}}\sqrt{E_0}\,(KT)^{3/2}\int_0^\infty t^{1/2}e^{-t}\,dt$$

$$\Gamma\left(\tfrac{3}{2}\right) = \Gamma\left(\tfrac{1}{2}+1\right) = \tfrac{1}{2}\Gamma\left(\tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2}$$

$$\boxed{\bar{\sigma}_{aT} = \frac{\sqrt{\pi}}{2}\cdot\sqrt{\frac{E_0}{KT}}\,\sigma_a(E_0) = \frac{\sqrt{\pi}}{2}\sqrt{\frac{T}{T_0}}\,\sigma_a(E_0)}$$

[$T_0 = 20^\circ C = 293K \Rightarrow E_0 = KT_0 = 0.025\,eV$ $\quad E_0 = \frac{1}{2}m_n v_0^2 \Rightarrow v_0 \simeq 2200\,m\,s^{-1}$]

$$\bar{\sigma}_{aT}(T) = \sqrt{\frac{T_0}{T}}\,\bar{\sigma}_{aT}(T_0)$$

$$\bar{\sigma}_{aT}(T_0) = \frac{\sqrt{\pi}}{2}\,\sigma_a(E_0)$$

$$\bar{\sigma}_{aT}(T) = g\sqrt{\frac{T_0}{T}}\,\bar{\sigma}_{aT}(T_0)$$

↳ non-$\frac{1}{v}$ correction / Westcott factor.

prob: 3.11, 3.11 E. Lewish (HW)

3.6. Infinite Medium Multiplication

Inf. med → no leakage

$$k_\infty = \frac{\text{production rate}}{\text{absorption rate}} = \frac{\int_0^\infty \nu\Sigma_f(E)\,\varphi(E)\,dE}{\int_0^\infty \Sigma_a(E)\,\varphi(E)\,dE}$$

$$= \frac{\nu\bar{\Sigma}_f\bar{\phi}}{\bar{\Sigma}_a\bar{\phi}} = \frac{\nu\bar{\Sigma}_f}{\bar{\Sigma}_a}$$

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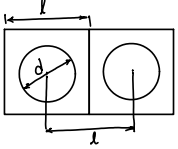
4. Power reactor cores

Wednesday, September 27, 2023 8:03 AM

Prob 4.1. Some Geometric Calculations in square & Hexagonal lattices

$\frac{V_f}{V_m} = \frac{1}{2}$ $d = 1.2$ cm $\Rightarrow r = \frac{d}{2}$



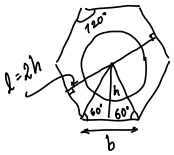
pitch, l – ?

$$\frac{V_f}{V_m} = \frac{\pi r^2 \cancel{h}}{l^2 \cancel{h} - \pi r^2 \cancel{h}} = \frac{\pi r^2}{l^2 - \pi r^2}$$

$$\Rightarrow l^2 = \frac{V_m}{V_f}\pi r^2 + \pi r^2 = \pi r^2\left(\frac{V_m}{V_f} + 1\right)$$

Square lattice

$$\Rightarrow \boxed{l = r\sqrt{\pi\left(\frac{V_m}{V_f} + 1\right)}}$$



$$\frac{V_f}{V_m} = \frac{\pi r^2 \cancel{H}}{A_{hex}\cancel{H} - \pi r^2 \cancel{H}}$$

$h = \frac{b}{2}\tan 60° = \frac{\sqrt{3}}{2}b$

$A_{hex} = 6 \cdot \frac{1}{2}bh = \frac{3\sqrt{3}}{2}b^2$

$l = 2h = \sqrt{3}\,b$

$$\Rightarrow \frac{V_f}{V_m} = \frac{\pi r^2}{\frac{3^{3/2}}{2}b^2 - \pi r^2}$$

$$\Rightarrow 3^{3/2}b^2 = \frac{V_m}{V_f}2\pi r^2 + 2\pi r^2$$

$$\Rightarrow b = 3^{-3/4} r\sqrt{2\pi\left(\frac{V_m}{V_f} + 1\right)}$$

$$\Rightarrow \boxed{l = \sqrt{3}b = 3^{-1/4} r\sqrt{2\pi\left(V_m/V_f + 1\right)}}$$

4.3. Fast Reactor Lattices

Assuming same flux $\varphi(E)$ everywhere in the reactor

$$k_\infty = \frac{\int_0^\infty \nu\Sigma_f(E)\,\varphi(E)\,dE}{\int_0^\infty \Sigma_a(E)\,\varphi(E)\,dE}$$

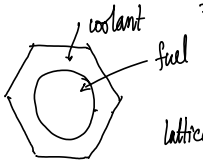
numerator → rate of neutron generation per cm³ per second

denominator → rate of neutron absorption per cm³ per second.

Fast reactor → $E_n \uparrow \Rightarrow \Sigma \downarrow \Rightarrow \lambda \uparrow$ compared to thermal.

$\varphi(E)$ - flat across fuel and coolant : $\varphi_f(E) \approx \varphi_c(E) \approx \varphi_{st}(E)$

fuel, coolant, structural materials



lattice cell of a fast reactor.

$$\Sigma_x(E) = \frac{V_f}{V}\Sigma_x^f(E) + \frac{V_c}{V}\Sigma_x^c(E) + \frac{V_{st}}{V}\Sigma_x^{st} = \sum_{y=f,c,st}\frac{V_y}{V}\Sigma_x^y$$

k_∞ ↓ $V = V_f + V_c + V_{st}$ – cell volume

$$k_\infty = \frac{\sum_{y=f,c,st}\int_0^\infty \frac{V_y}{V}\nu_y\Sigma_f^y(E)\,\varphi(E)\,dE}{\sum_{y=f,c,st}\int_0^\infty \frac{V_y}{V}\Sigma_a^y(E)\,\varphi(E)\,dE} \overset{\Sigma_f^{c,st}=0}{=} \frac{V_f\int_0^\infty \nu\Sigma_f^f(E)\,\varphi(E)\,dE}{\sum_{y=f,c,st}\int_0^\infty \frac{V_y}{V}\Sigma_a^y(E)\,\varphi(E)\,dE}$$

$\phi = \int_0^\infty \varphi(E)\,dE$ – total flux

$$\bar{\Sigma}_x^y = \frac{\int_0^\infty \Sigma_x^y(E)\,\varphi(E)\,dE}{\underbrace{\int_0^\infty \varphi(E)\,dE}_{\phi}} \qquad y = f, c, st.$$

∞

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$$\bar{\Sigma}_x^g = \frac{\int_0^\infty \Sigma_x^g(E)\,\varphi(E)\,dE}{\int_0^\infty \varphi(E)\,dE} \qquad g = f, c, ST$$

$$\Rightarrow \int_0^\infty \Sigma_x^g(E)\,\varphi(E)\,dE = \bar{\Sigma}_x^g \phi$$

$$k_\infty = \frac{V_f \nu \bar{\Sigma}_f^f}{V_f \bar{\Sigma}_a^f + V_c \bar{\Sigma}_a^c + V_{st} \bar{\Sigma}_a^{st}} \qquad k_\infty = \frac{\nu \bar{\Sigma}_f}{\bar{\Sigma}_a} = \frac{\nu \frac{V_f}{V} \bar{\Sigma}_f^f}{\frac{V_f}{V} \bar{\Sigma}_a^f + \frac{V_c}{V} \bar{\Sigma}_a^c}$$

enrichment consideration: fuel < fissile / fertile

$$N_f = N_{fi} + N_{fe} \qquad \tilde{e} = \frac{N_{fi}}{N_f} \text{ - enrichment}$$

$$\bar{\Sigma}_x^g = N_g \bar{\sigma}_x^g \quad , \quad \bar{\sigma}_x^g = \frac{\int_0^\infty \sigma_x^g(E)\,\varphi(E)\,dE}{\int_0^\infty \varphi(E)\,dE}$$

fuel: $\bar{\Sigma}_x^f = \bar{\Sigma}_x^{fi} + \bar{\Sigma}_x^{fe}$

$$\Rightarrow N_f \bar{\sigma}_x^f = N_{fi} \bar{\sigma}_x^{fi} + N_{fe} \bar{\sigma}_x^{fe} = \tilde{e}\,\sigma_1^{fi} + (1-\tilde{e})\,\bar{\sigma}_x^{fe}$$

$$k_\infty = \frac{V_f \nu \bar{\Sigma}_f^f}{V_f \bar{\Sigma}_a^f + V_c \bar{\Sigma}_a^c + V_{st} \bar{\Sigma}_a^{st}} = \frac{V_f N_f [\tilde{e}\,\nu^{fi} \bar{\sigma}_f^{fi} + (1-\tilde{e})\,\nu^{fe} \bar{\sigma}_f^{fe}]}{V_f N_f [\tilde{e}\,\bar{\sigma}_a^{fi} + (1-\tilde{e})\,\bar{\sigma}_a^{fe}] + V_c N_c \bar{\sigma}_a^c + V_{st} N_{st} \bar{\sigma}_a^{st}}$$

Problem 4.3

$V = V_f + V_c + V_{st}$; $V_f = V_{PuO_2} + V_{UO_2}$

k_∞ - ? fuel mass fraction ?

$$\beta_{PuO_2} = \frac{V_{PuO_2}}{V_f} = 0.15$$

$$\beta_{UO_2} = \frac{V_{UO_2}}{V_f} = (1-\beta_{PuO_2}) = 85\%$$

$$\beta_f = \frac{V_f}{V} = 30\%$$

$$\beta_c = \frac{V_c}{V} = 50\%$$

$$\beta_{st} = \frac{V_{st}}{V} = 20\%$$

$$k_\infty = \frac{\bar{\nu}\bar{\Sigma}_f}{\bar{\Sigma}_a}$$

$^{239}_{94}Pu$ $^{238}_{92}U$ $\qquad \Sigma = \frac{\rho N_A}{A}\sigma$

fuel — fission: $\bar{\nu}\bar{\Sigma}_f^f = \beta_{PuO_2} \cdot \beta_f \cdot \nu_{49} \cdot \bar{\Sigma}_f^{PuO_2} + \beta_{UO_2} \cdot \beta_f \cdot \nu_{28} \bar{\Sigma}_f^{UO_2}$

$$\frac{\rho_{PuO_2} N_A}{A_{PuO_2}} \bar{\sigma}_f^{49} \qquad \frac{\rho_{UO_2} N_A}{A_{UO_2}} \bar{\sigma}_f^{28}$$

absorption: $\bar{\Sigma}_a^f = \beta_{PuO_2} \cdot \beta_f \cdot \frac{\rho_{PuO_2} N_A}{A_{PuO_2}} \bar{\sigma}_a^{49} + \beta_{UO_2} \cdot \beta_f \cdot \frac{\rho_{UO_2} N_A}{A_{UO_2}} \bar{\sigma}_a^{28}$

coolant: $\bar{\Sigma}_a^c = \beta_c \cdot \frac{\rho_{Na} N_A}{A_{Na}} \bar{\sigma}_a^{Na}$ st: $\Sigma_a^{st} = \beta_{st} \cdot \frac{\rho_{Fe} N_A}{A_{Fe}} \bar{\sigma}_a^{Fe}$

$$k_\infty = \frac{\beta_{PuO_2} \cdot \beta_f \cdot \nu_{49} \cdot \frac{\rho_{PuO_2}}{A_{PuO_2}} \cdot \bar{\sigma}_f^{49} + \beta_{UO_2} \cdot \beta_f \cdot \nu_{28} \frac{\rho_{UO_2}}{A_{UO_2}} \bar{\sigma}_f^{28}}{\left(\beta_{PuO_2} \cdot \beta_f \cdot \frac{\rho_{PuO_2}}{A_{PuO_2}} \bar{\sigma}_a^{49} + \beta_{UO_2} \cdot \beta_f \cdot \frac{\rho_{UO_2}}{A_{UO_2}} \bar{\sigma}_a^{28}\right) + \beta_c \frac{\rho_{Na}}{A_{Na}} \bar{\sigma}_a^{Na} + \beta_{st} \frac{\rho_{Fe}}{A_{Fe}} \bar{\sigma}_a^{Fe}}$$

$A_{PuO_2} = 239 + 2 \times 16 =$ $\qquad A_{UO_2} = 238 + 2 \times 16 =$

NB: 4.4. Thermal Reactor Lattices: Ref. to the book + Problem 4.6, 4.7, 4.10.

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5. Reactor Kinetics

Sunday, January 14, 2024 9:11 PM

5.1. Introduction

subject matter: Time dependent behavior of neutron chain reaction.

simplifications: to focus on t variable, we assume that parameters are

① Energy averaged (E) - single energy group

② Spatially averaged ($\vec{r}$) - point kinetics.

outline

- Balance equations
  - non-multiplying system
  - multiplying system
- leakage effect
- effect of delayed neutrons.

Problem. 5.2

$f_p^{49} = 30\%$

Prelude

- Fission rate, $R_f = \frac{P}{E_f}$
- Fission neutron, $N_n = \nu R_f$
- Delayed neutrons, $N_{dn} = \beta N_n = \beta \nu \frac{P}{E_f}$
- Prompt neutrons, $N_{pn} = (1-\beta) N_n$

$P_{49} = f_p^{49} \cdot P$

$\nu_{25} = 2.43$, $\beta_{25} = 0.65\%$

$\nu_{49} = 2.90$, $\beta_{49} = 0.21\%$

$\beta_{BOL} = \beta_{25} = 0.65\%$

$P_{25} = (1 - f_p^{49}) P$

$P_{49} = f_p^{49} P$

$$\beta_{EOL} = \frac{N_{dn}^{EOL}}{N_n^{EOL}} = \frac{\beta_{25} \cdot \nu_{25} \cdot \frac{P_{25}}{E_f} + \beta_{49} \cdot \nu_{49} \frac{P_{49}}{E_f}}{\nu_{25} \frac{P_{25}}{E_f} + \nu_{49} \frac{P_{49}}{E_f}}$$

$f_p = 0.3$

$$\beta_{EOL} = \frac{\beta_{25} \nu_{25} (1 - f_p^{49}) + \beta_{49} \cdot \nu_{49} \cdot f_p^{49}}{\nu_{25} (1 - f_p^{25}) + \nu_{49} f_p^{49}} = 0.501\%$$

$$\Delta\beta = \frac{\beta_{EOL} - \beta_{BOL}}{\beta_{BOL}} \approx -23\%$$

5.2. Neutron Balance equations

$n(t)$ - total number of neutrons at time t

$\bar{v}$ - average neutron speed

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Σ_x - energy & space averaged XS

5.2.1. Infinite medium non multiplying Systems

$$\frac{d}{dt}n(t) = S(t) - A(t)$$

$S(t)$: source production rate; $A(t)$: neutron absorption rate

$$A(t) = \Sigma_a \bar{v}\, n(t)$$

$\Sigma_a \bar{v}$ = frequency of collision

$[S(t)] = \frac{\#\text{ neutron}}{s}$ - source strength.

$\frac{1}{\Sigma_a \bar{v}}$ - lifetime

$$\boxed{\frac{dn}{dt} = S(t) - \Sigma_a \bar{v}\, n(t)}$$

$[n(t=0) = n_0 \quad S(t) = 0 \quad t>0]$

$$\frac{dn}{dt} = -\Sigma_a \bar{v}\, n(t) \Rightarrow \boxed{n(t) = n_0 e^{-\Sigma_a \bar{v} t}}$$

average neutron lifetime, $\bar{t} = \frac{\int_0^\infty t\, n(t)\, dt}{\int_0^\infty n(t)\, dt} = \frac{\int_0^\infty t\, e^{-\Sigma_a \bar{v} t}\, dt}{\int_0^\infty e^{-\Sigma_a \bar{v} t}\, dt}$

$$\bar{t} = \frac{\frac{1}{(\Sigma_a \bar{v})^2}\int_0^\infty (\Sigma_a \bar{v} t)\, e^{-\Sigma_a \bar{v} t}\cdot d(\Sigma_a \bar{v} t)}{\frac{1}{\Sigma_a \bar{v}}\, e^{-\Sigma_a \bar{v} t}\Big|_0^\infty} = \frac{1}{\Sigma_a \bar{v}} \underbrace{\int_0^\infty u\, e^{-u}\, du}_{\Gamma(2) = 1\cdot\Gamma(1) = 1}$$

$$l_\infty = \frac{1}{\Sigma_a \bar{v}}$$ - average lifetime in infinite medium.

Bal. eqn

$$\frac{dn}{dt} + \frac{n(t)}{l_\infty} = S(t)$$

$\times e^{t/l_\infty} \Rightarrow \frac{dn}{dt}\cdot e^{t/l_\infty} + \frac{n(t)}{l_\infty} e^{t/l_\infty} = S(t) e^{t/l_\infty}$

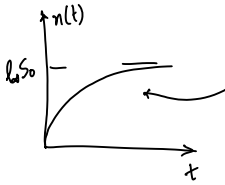
$\Rightarrow \frac{d}{dt}\left(n(t) e^{t/l_\infty}\right) = S(t) e^{t/l_\infty}$

$\Rightarrow \int_0^t d\left(n(t') e^{t'/l_\infty}\right) = \int_0^t S(t') e^{t'/l_\infty} dt'$ $\quad t'$ – dummy variable

$\Rightarrow n(t) e^{t/l_\infty} - n(0) = \int_0^t S(t') e^{t'/l_\infty} dt'$

$\Rightarrow \boxed{n(t) = n(0) e^{-t/l_\infty} + \int_0^t S(t') e^{\frac{t'-t}{l_\infty}} dt'}$ – general solution.

particular case: $S(t) = S_0 = \text{const}$; $n(0) = 0$



$$n(t) = S_0\, e^{\frac{t'-t}{l_\infty}}\Big|_0^t = \boxed{l_\infty S_0 \left(1 - e^{-t/l_\infty}\right) = n(t)}$$

$n(\infty) = l_\infty S_0$ - equilibrium.

Prob. 5.1. (l_∞ – in Thermal & Fast Reactors) $\quad A(t) = \Sigma_a \bar{v}\, n(t)$

a) $E_T = 0.0253\,eV \quad \Sigma_{aT}^c = 0.273 \times 10^{-3}\,cm^{-1}$

$$\bar{v}_T = \sqrt{\frac{2E_T}{m_n}} \approx 2200\ m/s \Rightarrow l_\infty^c = \frac{1}{\bar{v}_T \Sigma_{aT}^c} = 0.017\ s$$

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b) $E_F = 1\,MeV$, $\Sigma_{aF}^{Fe} = 0.738 \times 10^{-2}\,cm^{-1}$

$\bar{v}_F = \sqrt{\frac{2E_F}{m_n}} \approx 1.38 \times 10^7 \frac{m}{s} \Rightarrow l_\infty^{Fe} = \frac{1}{\bar{v}_F \Sigma_{aF}^{Fe}} = 0.98 \times 10^{-6} s$

5.2.2. Intinite medium multiplying system

$\frac{d}{dt} n(t) = S(t) + F(t) - A(t)$ (source, Fission, Absorption) $\quad F(t) = \nu \Sigma_f \bar{v} n(t)$ ($\Sigma_x \bar{v}$ - frequency of x-type reaction)

$\Rightarrow \frac{dn}{dt} = S(t) + \nu \Sigma_f \bar{v} n(t) - \Sigma_a \bar{v} n(t)$

neutron lifetime, $l_\infty = \frac{1}{\Sigma_a \bar{v}} \Rightarrow \Sigma_a = \frac{1}{\bar{v} l_\infty}$

multiplication, $k_\infty = \frac{\nu \Sigma_f}{\Sigma_a} \Rightarrow \nu \Sigma_f = \frac{k_\infty}{\bar{v} l_\infty}$

$\Rightarrow \frac{dn}{dt} = S(t) + \frac{k_\infty \cancel{\bar{v}} n(t)}{\cancel{\bar{v}} l_\infty} - \frac{n(t) \cancel{\bar{v}}}{\cancel{\bar{v}} l_\infty}$

no ext. source, $S(t) = 0 \Rightarrow \frac{dn}{dt} = \frac{k_\infty - 1}{l_\infty} n(t) + \frac{k_\infty - 1}{l_\infty} n(t)$

- critical: $k_\infty = 1 \Rightarrow \frac{dn}{dt} = 0 \Rightarrow n = const.$
- subcritical: $k_\infty < 1 \Rightarrow \frac{dn}{dt} < 0 \Rightarrow n(t) \downarrow$
- supercritical: $k_\infty > 1 \Rightarrow \frac{dn}{dt} > 0 \Rightarrow n(t) \uparrow$

5.2.3. Finite multiplying system.

$\frac{d}{dt} n(t) = S(t) + F(t) - A(t) - L(t)$ (source, Fission, Absorption, Leakage)

$L(t) \propto \Sigma_a \bar{v} n(t) \Rightarrow L(t) = \Gamma \cdot \Sigma_a \bar{v} n(t)$ (Γ: proportionality constant.)

Neutron loss rate $= A(t) + L(t)$

$\Rightarrow$ Leakage probability, $P_L = \frac{\Gamma \Sigma_a \bar{v} n(t)}{\Sigma_a \bar{v} n(t) + \Gamma \Sigma_a \bar{v} n(t)} = \frac{\Gamma}{1+\Gamma}$

$\Rightarrow$ Nonleakage prob. $P_{NL} = 1 - P_L = 1 - \frac{\Gamma}{1+\Gamma} = \frac{1}{1+\Gamma}$

$\Rightarrow \Gamma = \frac{1}{P_{NL}} - 1$

Balance eqn

$\Rightarrow \frac{dn}{dt} = S(t) + \nu \Sigma_f \bar{v} n(t) - \Sigma_a \bar{v} n(t) - \left(\frac{1}{P_{NL}} - 1\right) \Sigma_a \bar{v} n(t) = S(t) + \nu \Sigma_f \bar{v} n(t) - \frac{\Sigma_a \bar{v} n(t)}{P_{NL}}$

($l_\infty = \frac{1}{\bar{v} \Sigma_a} \Rightarrow \Sigma_a = \frac{1}{\bar{v} l_\infty}$; $k_\infty = \frac{\nu \Sigma_f}{\Sigma_a} \Rightarrow \nu \Sigma_f = \frac{k_\infty}{\bar{v} l_\infty}$)

$\Rightarrow \frac{dn}{dt} = S(t) + \frac{P_{NL} k_\infty - 1}{P_{NL} l_\infty} n(t)$

$\Rightarrow \boxed{\frac{dn}{dt} = S(t) + \frac{k-1}{l} n(t)}$

- $k_\infty P_{NL} = k$ - finite med. multiplication
- $P_{NL} \cdot l_\infty = l$ - finite med. lifetime.

5.3. Multiplying System Behavior.

① $S(t) = 0 \Rightarrow \frac{dn}{dt} = \frac{k-1}{l} n(t) \leftarrow$

$n(t)$ ... t ... $n(t)$ ...

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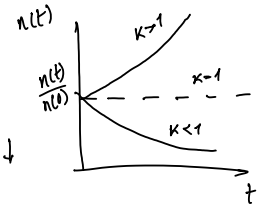
$$\Rightarrow \int_{n(0)}^{n(t)} \frac{dn'}{n'} = \int_0^t \frac{k-1}{l}\,dt' \Rightarrow \ln\frac{n(t)}{n(0)} = \frac{k-1}{l}t$$

$$\Rightarrow \boxed{n(t) = n(0)\exp\left[\frac{k-1}{l}t\right]}$$

critical: $k=1 \Rightarrow \frac{dn}{dt}=0 \Rightarrow n(t)=n(0)$

supercritical: $k>1 \Rightarrow \frac{dn}{dt}>0 \Rightarrow n(t) = n(0)\exp\left[\frac{k-1}{l}t\right] \uparrow$

subcritical: $k<1 \Rightarrow \frac{dn}{dt}<0 \Rightarrow n(t) = n(0)\exp\left[-\frac{|k-1|}{l}t\right] \downarrow$ ($k-1<0$)



② $S(t) = S_0 = \text{const} \neq 0$

$$\frac{dn}{dt} = S_0 + \frac{k-1}{l}n(t) \quad \times \exp\left[-\frac{k-1}{l}t\right]$$

$$\Rightarrow \frac{dn}{dt}e^{-\frac{k-1}{l}t} - \frac{k-1}{l}n(t)e^{-\frac{k-1}{l}t} = S_0 \cdot e^{-\frac{k-1}{l}t}$$

chain rule $\Rightarrow \frac{d}{dt}\left[n(t)e^{-\frac{k-1}{l}t}\right] = S_0 e^{-\frac{k-1}{l}t}$

$$\Rightarrow \int_{n(0)}^{n(t)} d\left[n(t')e^{-\frac{k-1}{l}t'}\right] = S_0\int_0^t e^{-\frac{k-1}{l}t'}dt' = \frac{lS_0}{k-1}e^{-\frac{k-1}{l}t'}\Big|_0^t$$

$$\Rightarrow n(t)e^{-\frac{k-1}{l}t} - n(0) = \frac{lS_0}{k-1}\left[1 - e^{-\frac{k-1}{l}t}\right]$$

$$\Rightarrow \boxed{n(t) = n(0)e^{\frac{k-1}{l}t} + \frac{lS_0}{k-1}\left[e^{\frac{k-1}{l}t} - 1\right]}$$

$n(0)=0 \Rightarrow n(t) = \frac{lS_0}{k-1}\left\{\exp\left[\frac{k-1}{l}t\right]-1\right\}$

$k>1$: $n(t)$ – exponentially growing function

$k<1$: $n(t) = \frac{lS_0}{1-k}\left(1 - e^{-\frac{1-k}{l}t}\right)$

assymptotically reaches equilibrium: $\boxed{n(\infty) = \frac{lS_0}{1-k}}$

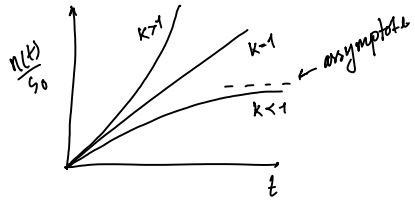
($e^x = 1 + x + x^2/2 + \dots$)

$k=1$: ($\frac{0}{0}$ form)

$$n(t) = \lim_{k\to 1}\frac{lS_0}{k-1}\left[\cancel{1} + \frac{k-1}{l}t + \frac{1}{2}\left(\frac{k-1}{l}t\right)^2 + \dots - \cancel{1}\right]$$

$$= \lim_{k\to 1} lS_0\left[\frac{1}{l} + \frac{1}{2}\left(\frac{k-1}{l}t\right) + \dots\right]$$

$\boxed{n(t) = S_0 t}$ ~ linearly increasing function.



Stationary system $\frac{dn}{dt} = 0 \Rightarrow \underbrace{S(t)}_{\geq 0} + \frac{k-1}{l}\underbrace{n(t)}_{n\geq 0} = 0$

two cases satisfy the condition:

① $k=1 \wedge S(t)=0$

② $k<1 \wedge S(t)>0$

$\wedge$ – and

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Problem 5.3

$n(0) = 0$ $\quad S(t) = \begin{cases} S_0 & 0 < t \le 1 \text{ min} \\ 0 & \text{otherwise.} \end{cases}$

a) $k < 1$: $n(t)$? $\quad 0 \le t < ?$ min

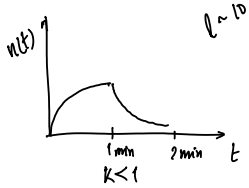
b) $k = 1$: $n(t)$ - ? $\quad 0 \le t < ?$ min

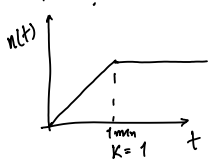
sketch

$$n(t) = n(0)\, e^{\frac{k-1}{l}t} + \frac{l S_0}{k-1}\left[e^{\frac{k-1}{l}t} - 1\right]$$

$l \sim 10^{-3}$ s

$\sim n(1\text{min})\, e^{-\frac{|k-1|}{l}t}$





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